The Islamic University of Gaza
Deanship of Graduate Studies
Faculty of Engineering
Electrical Engineering Department

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عمادة الدراسات العليا
كلية الهندسة
قسم الهندسة الكهربائية

Optimal Force Distribution in Multi-finger Dexterous Hands for Robotic Systems with Uncertainty

By

Eng. Raed Abdul Rahman Al Habibi

Advisor

Prof. Mohammed Hussein

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Abstract

Robotics is the field of technology that deals with the design, construction, operation, structural disposition, manufacture. The main advantage of robots and computer systems are their control, sensory feedback, and information processing. These technologies deal with automated machines that can take the place of humans in dangerous or manufacturing processes.

During the last two decades several techniques are used to design and control the movement of the robotics hand grasping .

Dexterous manipulation by multi-fingered robot hands is one of the challenging and complicated problems in robotics. In the early research on hand robots, the multi-joint-fingered models which are similar to the human hand in appearance were primarily presented. By increasing the interest in the dexterous manipulation, the researches on the various sensors and control methods have been required.

Many of the proposed method used to control the robotics hand grasping without measuring the forces and the torques of the fingers and without determining the optimal position of the body .

The proposed method in this thesis uses the linear programming (LP) and Semidefinite programming (SDP) methods to measure the optimal force for robotic grasping hand for four fingers.

In addition, we proposed a new method to based on uncertainty theorem; moreover, our method is used to measure the upper and lower forces then, choose the optimal one using the software package MATLAB.

To demonstrate the effectiveness of the presented techniques several optimization examples are solved.

ملخص :

حساب القوى الخاصة بيد الرجل الالي بطريقة البرمجة الخطية والشبه مؤكدة

الروبوتات هو حقل من حقول التكنولوجيا تتعامل مع تصميم وبناء وتشغيل والتصرف الهيكلي، والتصنيع. والميزة الرئيسية لأجهزة الروبوت وأنظمة الكمبيوتر وسيطرتهم، وردود الفعل الحسية، ومعالجة المعلومات. هذه التقنيات تتعامل مع الآلات الأوتوماتيكية التي يمكن أن تحل محل البشر في العمليات الخطرة أو الصناعات التحويلية.

خلال العقدين الماضيين استخدمت تقنيات عدة للتصميم والتحكم في استيعاب حركة اليد الروبوتية.

التلاعب الحاذق في يد الروبوت متعدد الأصابع هي مشكلة من المشاكل الصعبة والمعقدة في مجال الروبوتات ، وعرضت في المقام الأول على نماذج متعددة من انظمة الايدي وتعدده الاصابع التي تشبه اليد البشرية من حيث المظهر. من خلال زيادة الاهتمام في التلاعب حاذق، وقد يطلب من الأبحاث على مختلف أجهزة الاستشعار وطرق المكافحة.

العديد من الطرق المقترحة تستخدم للسيطرة على يد الروبوتات دون قياس استيعاب القوى وعزم الدوران للأصابع ودون تحديد الموقف الأمثل من الجسم.

الطريقة المقترحة في هذه الأطروحة يستخدم البرمجة الخطية والبرمجة شبه المؤكدة وهي طرق لقياس قوة الأمثل لاستيعاب اليد الروبوتية لأربعة أصابع.

وبالإضافة إلى ذلك، اقترحنا طريقة جديدة قائمة على نظرية عدم اليقين، وعلاوة على ذلك، يتم استخدام الطريقة المقترحة لقياس القوى العلوية والسفلية ثم اختيار القوة الأمثل او المثلى باستخدام برنامج الماتلاب.

للتدليل على فعالية التقنيات المعروضة تم حل مجموعة من الأمثلة.

DEDICATION

To my parents,

My wife and my sweet kids,

Maha , Layan and Abdul Rahman

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First and forever, all praise and thanks for Allah, who gave me the strength, and patience to carry out this work in this good manner.

I would like to deeply thank my supervisors Prof. Mohammed Hussein for their assistance, guidance, support, patience, and encouragement.

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Finally, great thanks to my family for their endless praying and continuous support.

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List of Abbreviations

μ_{ti}	Constant between the Torsion and Shear Limits.
CSM	Constraint Stabilization Method.
C_i	Contact Force.
1- DOF	Degree of Freedom.
f_{ext}	External Force Exerted on The Object.
ϕ	Free Parameter Vector.
μ_i	Friction Coefficient.
τ_{ij}	Joint Torques.
α_i	Linear Model or Elliptical Approximation
β_i	Linear Model or Elliptical Approximation
LP	Linear Programming.
W^+	Moore-Penrose Pseudo Inverse.
m	Number of Fingers Grasping.
SDP	Semidefinite Programming.
Σ	Singular Value Decomposition.
SVD	Singular Value Decomposition.
V_η	The Matrix Formed Using the Last 6 Columns of V.
ϕ*	The Minimizer of the LP and SDP.
Δ	Uncertainty Interval.
ULP	Uncertainty Linear Programming.
USDP	Uncertainty Semidefinite Programming.
σ	Uncertainty Value.

Chapter One

Introduction

1.1 Background

Robots are systems that consolidate mechanical parts and electronic controls to perform tasks. As robotic systems have evolved, the capabilities of robotic systems have skyrocketed, and robots are used in many industries to perform multiple processes. Robots are used in motor car assembly lines to create spot welds, operating rooms to perform surgery, and in space to move equipment into place. Traditional robotic systems integrated mechanical components and electronic control systems to perform a set series of tasks, such as move a part, perform an action, and repeat the task over again.

The concept and creation of machines that could operate autonomously dates back to classical times, that robotics have been often seen to mimic human behavior, and often manage tasks in a similar fashion. Today, robotics is a rapidly growing field, as we continue to research, design, and build new robots that serve various practical purposes, whether domestically, commercially, or militarily. Many robots do jobs that are hazardous to people such as defusing bombs, exploring shipwrecks, and mines.

Dexterous manipulation by multi-fingered robot hands is one of the challenging and complicated problems in robotics. In the early research on hand robots, the multi-joint-fingered models which are similar to the human hand in appearance were primarily presented. By increasing the interest in the dexterous manipulation, the researches on the various sensors and control methods have been required.

In this thesis, the proposed method will study the force and the torque that apply on static body then find the optimal values based on linear programming (LP) and Semidefinite programming (SDP), with incorporation uncertainty in the design model.

1.2 Motivations

Many researchers design dexterous manipulation depending on different methods such as Buss [1] that proposed optimization technique based on Riccati equation and Newton-Raphson-flows which is considered as complex way. Song [2] also used force angle optimization and position regulation; but the objects are smooth surface and in a horizontal plane.

In this thesis, we propose a new and effective technique depending on LP and SDP by considering the uncertainty forces which make dexterous fingers more robust.

1.3 Thesis Objectives

Objectives of this thesis can be summarized as follows:

- To understand how robotics can carry or bush any object.
- To know how many forces and forces coordination that affects on the objects.
- To check about how many fingers can we used to not smash the object when we catch it with the robot fingers.
- To apply the uncertainty control in order to have robust system.

1.4 Statement of Problem

The main problem of the dexterous Hands for Robotic Systems can be summarized in the following as in Eq. (1.1) and (1.2). Moreover, the equations can be divided into two systems the first is LP system and the second is SDP system.

- LP

$$p(C) = w^T C \qquad (1.1a)$$

$$Ac \geq 0 \qquad (1.1b)$$

$$Wc = -f_{ext} \qquad (1.1c)$$

Where:

W: Matrix whose columns comprise the directions of the *m* contact forces.
C: Optimal Contact Force.
f_{ext}: External Force.
P(c): The Objective Function.
Ac: Friction-Force Constraints.

- SDP

$$p = w^T c \qquad (1.2a)$$

$$Wc = -f_{ext} \qquad (1.2b)$$

$$P(c) \geq 0 \qquad (1.2c)$$

Where:

W: Matrix whose columns comprise the directions of the *m* contact forces.
C: Optimal Contact Force.
f_{ext}: External Force.
P(c): The Linear inequality constraints.

The task in this thesis will calculate the uncertain forces by applying uncertain LP and uncertain SDP techniques.

1.5 Literature Review

Many researchers have been tried to design and optimize of robotics finger using different approaches and methods as follows:

- In 2011, Zhan and et al. [3] design and optimized a robotic finger by 1- DOF three finger phalanges robotic finger , link driven finger with DOF and they made it with Grasp simulation; but their method was difficult.

- In 2011, Sasaki and et al. [4] this research utilize force-distribution-based evaluation of product design suitable for dynamically dexterous human hand manipulation by using gloves type hand poster measuring instrument and physics simulation with camera interfacing; but they used an expensive camera.

- In 2011, Hashimoto and et al. [5] this research employed force distribution measurement to investigate dexterous manipulation of hand model , dynamics space, and data gloves so they developed a prototype system for evaluation of manipulation that consists of computer running the dynamic simulation and data gloves for a human operator to operate the hand model but they also used an expensive camera.

- In 2011, Song and et al. [2] this article presents Stable Grasping control method of dual fingered robotic hands from force angle optimization and position regulation, it's a control method for stable grasping of an object with optimal force angle and the position regulation of the object, the numerical simulations for the finger links have been performed to demonstrate the effectiveness of the proposed controller using CSM.

- In 2010, Al-Gallaf [6] this article presents A learning rule-based robotics hand optimal force closure by using Neuro-fuzzy for four fingers robotics hand dynamic and kinematics relations using an artificial neural network.

- In 2010, Sugaiwa and et al. [7] this research employed a methodology for setting grasping force for picking up an object with unknown weight, friction, and stiffness, this methodology gives priority to avoiding dropping the object characteristics and priority to avoiding dropping the object first, squashing it second, and grasping it with excessive force and all three method produce motion that creates a discrete change of the object behavior.

- In 2008, Liu and et al. [8] this research is take on an optimal method to determine the parameters of anthropomorphic finger based on four-link mechanism by using existing vector kinematics based optimal method and finding an alternative approach based on geometry kinematics.

- In 2007, Antoniou and et al. [9] this book takes a practical optimization algorithms and engineering applications in many ways and have an examples about distribution in multi-finger dexterous hands for robotic system they test many control system like LP and SDP on the robotics and are concerned with constrained optimization methods. They are introduces the fundamentals of constrained optimization. The concept of Lagrange multipliers, the first-order necessary conditions known as Karush-Kuhn-Tucker conditions, and the duality principle of convex programming are addressed in detail and are illustrated by many examples. They concerned with LP problems, examines several applications of constrained optimization for the design of digital filters, for the control of dynamic systems, for evaluating the force distribution in robotic systems, and in multiuser detection for wireless communication systems; they only measured the forces with LP and SDP.

- In 1996, Buss and et al. [1] this research is take on a several forms of constrained gradient flows are developed for point contact and soft-finger contact friction models. They find that friction force limit constraints and force balancing constraints are equivalent to the positive definiteness of a certain matrix subject to linear constraints. Based on this observation, they formulate the task of grasping force optimization as an optimization problem on the smooth manifold of linearly constrained positive definite matrices for which there are known globally exponentially convergent solutions via gradient flows. There are a number of versions depending on the Riemannian metric chosen, each with its advantages. Schemes involving second derivative information for quadratic convergence are also studied. The physical meaning of the cost index used for the gradient flows is discussed in the context of grasping force optimization. A discretized version for real-time applicability is presented. Numerical examples demonstrate the simplicity, the good numerical properties, and optimality of the approach; they used the force balance constrains.

- In 2011, Hussein [10] This research employed a methodology of robust stability of an uncertain three dimensional (3-D) system using existence MATLAB convex hull functions. Hence, the uncertain model of plant will be simulated by INTLAB Toolbox; furthermore, the root loci of the characteristic polynomials of the convex hull are obtained to judge whether the uncertain system is stable or not. A design third order example for uncertain parameters is given to validate the proposed approach. This research tested the robust stability of an interval 3x3 matrix by the implementation of Printer Belt-Drive System. An efficient and enhanced algorithm was introduced and improved for this purpose. This algorithm can be easily extended to deal with higher order matrices (n-dimensional system) without a very large increase of processing time.

- In 2010, Hussein [11] This research presents an efficient computational method for generating a plot for the bounds of eigenvalues of the entire family of interval matrices through a simple and efficient algorithm. The convex-hull technique is utilized and incorporated to find the smallest convex polygon containing all characteristic polynomial points. This method requires fewer computations. Finally, an electrical circuit application was used to examine and evaluate the proposed algorithm, showing significant results. Also, it is hoped that this research will open the door towards future work in conjunction with current related research to consider higher dimension problems through the investigation of the possibility of developing a new convex hull algorithm that can accommodate higher dimensional problems, and yet can be implemented with parallel and distributed algorithms with the utilization of supercomputers to speed up the process of an excessive computational process involved in the higher dimension problems.

- In 2010, Hussein [12] M.T. Hussein, Lecture Notes for Control of Uncertain Systems Course Faculty of Engineering, IUG this course interview control engineering and control design , robust specifications, actuator technology, framework for control system architecture, Hansen inverse method, stability and performance robustance, and I am apply the roles of the uncertainty and interval arithmetic's on this system.

- I have emailed prof. Andreas Antoniou and prof. Wu-Sheng Lu the authors of compilation Practical Optimization Algorithms and Engineering Applications, Department of Electrical and Computer Engineering University of Victoria, Canada [9] and they send to me a recommendation and laudation about using uncertainty control system on robotics hand (see Appendix)

1.6 Thesis Contributions

The basic idea in our research will be based on the following steps:

- Formulation of cost function and constrains.
- Simulation environment to test the cost function with simple optimization method.
- Study different modern optimization method to find the most suitable techniques to solve the Multi-finger dexterous hands.
- Comparison with these method to identify the advantages and disadvantage for each method is simplicity cost implantation and feasibility .
- After selecting suitable advance method we will test the system under the simulation environment using different objectives, different finger arrangement and different friction coefficient in static way.
- Apply the uncertain LP technique and uncertain SDP technique.

1.7 Thesis Organization

The thesis is organized as follows: chapter two introduces the optimal distribution force for robotics system, chapter three presents the solution of optimal force distribution problem by using LP, chapter four presents solution of optimal force distribution problem by using SDP, and finally chapter five covers the conclusion, and future work of the research as shown in Figure (1.1).

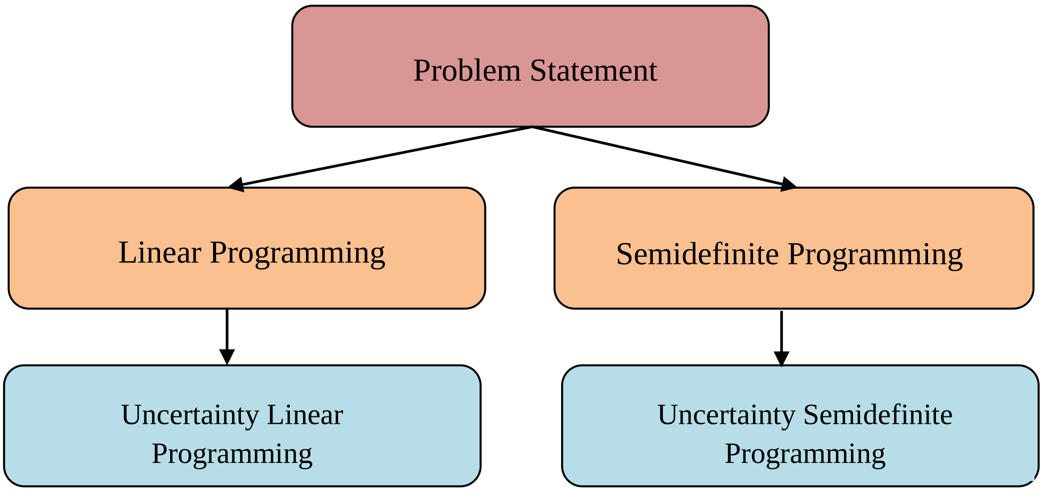

Figure (1.1) Organization Thesis Graph

Chapter Two

Introduction to Optimal Force Distribution for Robotic Systems

2.1 Introduction

Because of their use in a wide variety of applications ranging from robotic surgery to space exploration, robotic systems with closed kinematic loops such as multiple manipulators handling a single workload, dexterous hands with fingers closed through the object grasped in Figure (2.1), and multilegged vehicles with kinematic chains closed through the body in Figure (2.2) have become an increasingly important subject of study in the past several years. An issue of central importance for this class of robotic systems is the force distribution that determines the joint torques and forces to generate the desired motion of the workload [9].

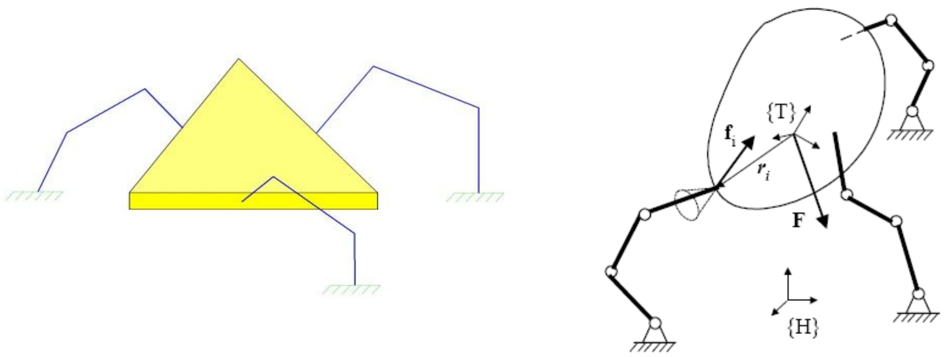

Figure (2.1) Three Coordinated Manipulators (Three-Finger Dexterous Hand) grasping an object.

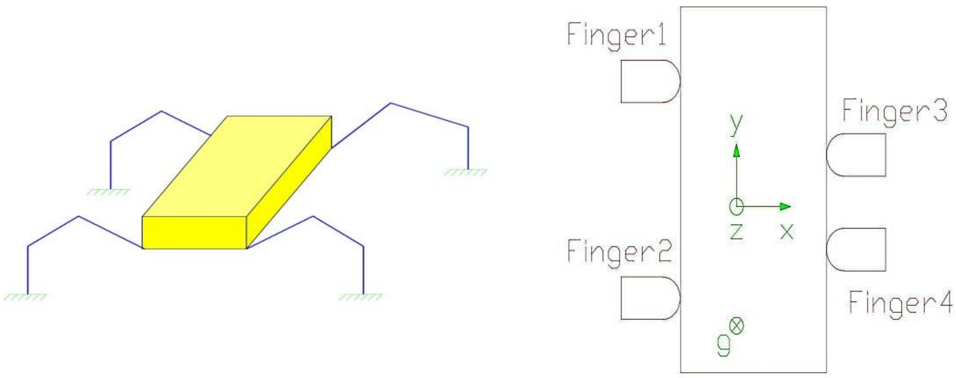

Figure (2.2) Multilegged Vehicle (Four-Finger Dexterous hand) Grasping an Object.

The force distribution problem for multifinger dexterous hands is described and two models for the contact forces are studied. The optimal force distribution problem is then formulated and solved using LP and SDP programming

2.2 Force distribution problem in multifinger dexterous hands

Consider a dexterous hand with m fingers grasping an object such as that depicted in Figure (2.3) for m = 3. The contact force C_i of the i^{th} finger is supplied by the finger's n_j joint torques τ_{ij} for j = 1, 2, . . . , n_j, and f_{ext} is an external force exerted on the object. The force distribution problem is to find

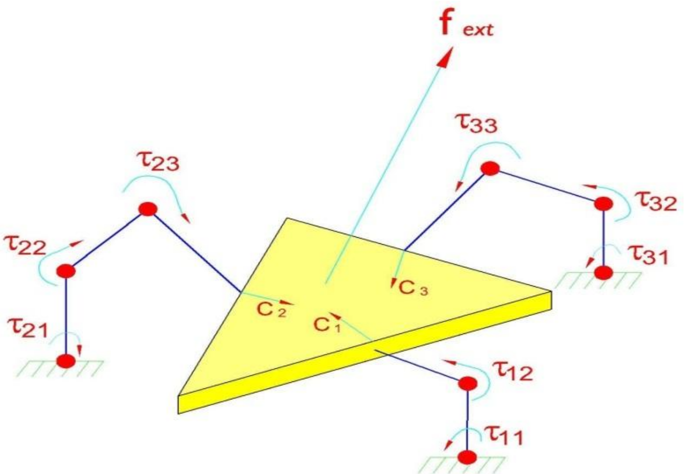

Figure (2.3) A three-finger hand grasping an object.

the contact forces C_i for $i = 1, 2, \ldots, m$ that would balance the external force $f_{ext} \in R^6$ so as to assure a stable grasp [13-17] . The dynamics of the system can be represented by the equation

$$Wc = -f_{ext} \tag{2.1}$$

Where:

f_{ext}: External Force.
C: Contact Force.
W: The direction of the m contact forces.

where c is a vector whose components are them contact m forces C_i for $1 \leq i \leq m$ and $W \in R^{6\times 3m}$ is a matrix whose columns comprise the directions of the m contact forces. The product vector Wc in Eq.(2.1) is a six-dimensional vector whose first three components represent the overall contact force and last three components represent the overall contact torque relative to a frame of reference with the center of mass of the object as its origin.

To maintain a stable grasp, the contact forces whose magnitudes are within the friction force limit must remain positive towards the object surface. There are two commonly used models to describe a contact force, namely, the point contact and soft-finger contact model. In the point-contact model, the contact force ci has three components, a component C_{i1} that is orthogonal and two components C_{i2} and C_{i3} that are tangential to the object surface as shown in Figure (2.4a). In the soft-finger contact model, c_i has an additional component C_{i4}, as shown in Figure (2.4b), that describes the torsional moment around the normal on the object surface.

Friction force plays an important role in stable grasping [13-17]. In a point-contact model, the friction constraint can be expressed as

$$\sqrt{C_{i2}^2 + C_{i3}^2} \leq \mu_i C_{i1} \tag{2.2}$$

where C_{i1} is the normal force component, C_{i2} and C_{i3} are the tangential components of the contact force C_i, and $\mu_i > 0$ denotes the friction coefficient at the contact point. It follows that for a given friction coefficient $\mu_i > 0$, the constraint in Eq. (2.2) describes a friction cone as illustrated in Figure (2.5).

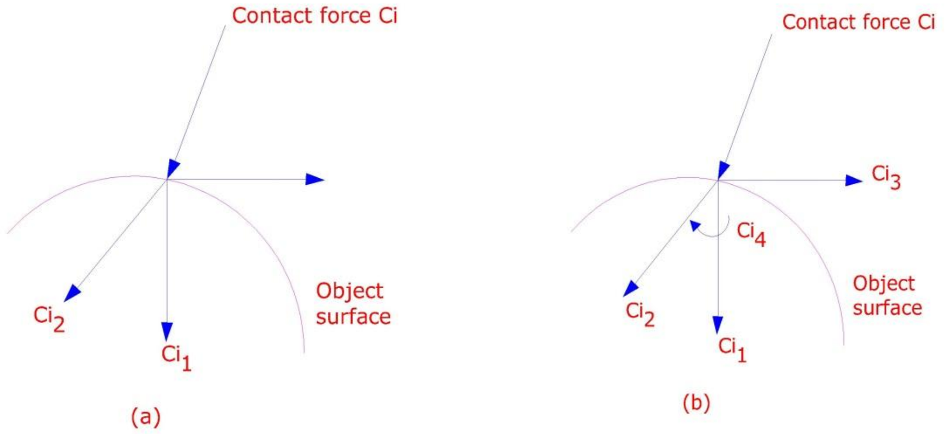

Figure (2.4) (a) Point-Contact Model, (b) Soft-Finger Contact Model.

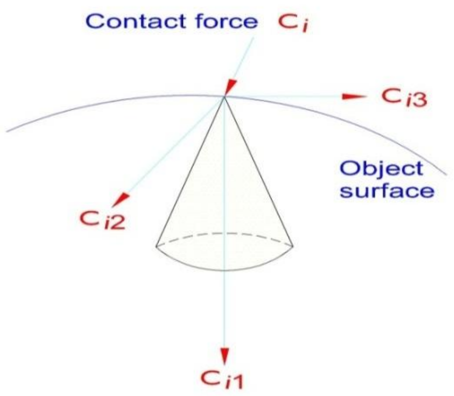

Figure (2.5) Friction cone as a constraint on contact force c_i.

Obviously, the friction force modeled by Eq. (2.2) is nonlinear: for a fixed μ_i and C_{i1}, the magnitude of the tangential force is constrained to within a circle of radius $\mu_i C_{i1}$. A linear constraint for the friction force can be obtained by approximating the circle with a square as shown in Figure (2.6). The approximation involved can be described in terms of the linear constraints [9].

$$C_{i1} \geq 0 \tag{2.3a}$$

$$-\frac{\mu_i}{\sqrt{2}} C_{i1} \leq C_{i2} \leq \frac{\mu_i}{\sqrt{2}} C_{i1} \tag{2.3b}$$

$$-\frac{\mu_i}{\sqrt{2}} C_{i1} \leq C_{i3} \leq \frac{\mu_i}{\sqrt{2}} C_{i1} \tag{2.3c}$$

The friction limits in a soft-finger contact model depend on both the torsion and shear forces, and can be described by a linear or an elliptical approximation. The linear model is given by .

$$\frac{1}{\mu_i} f_t + \frac{1}{\hat{\mu}_{ti}} |C_{i4}| \leq C_{i1} \tag{2.4}$$

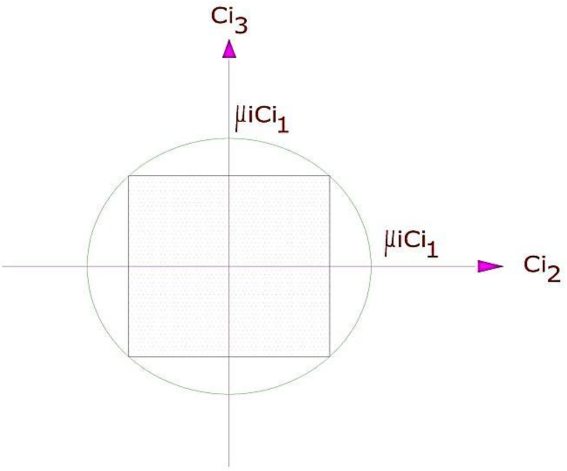

Figure (2.6) Linear approximation for friction cone constraint.

where $\hat{\mu}_{ti}$ is a constant between the torsion and shear limits, μ_i is the tangential friction coefficient, and $f_t = \sqrt{C_{i2}^2 + C_{i3}^2}$. The elliptical model, on the other hand, is described by

$$C_{i1} \geq 0 \tag{2.5a}$$

$$\frac{1}{\mu_i}(C_{i2}^2 + C_{i3}^2) + \frac{1}{\mu_{ti}} C_{i4}^2 \leq C_{i1}^2 \tag{2.5b}$$

where $\hat{\mu}_{ti}$ is a constant.

2.3 Uncertainty Control System

2.3.1. Interval Arithmetic

Interval arithmetic, interval mathematics, interval analysis, or interval computation, is a method developed by mathematicians since as an approach to putting bounds on rounding errors and measurement errors in mathematical computation and thus developing numerical methods that yield reliable results. Very simply put, it represents each value as a range of possibilities. Whereas classical arithmetic defines operations on individual numbers, interval arithmetic defines a set of operations on intervals This article is about intervals of real numbers and other totally ordered sets. For the most general definition, see partially ordered set.

In mathematics, a (real) interval is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set. For example, the set of all numbers x satisfying $0 \leq x \leq 1$ is an interval which contains 0 and 1, as well as all numbers between them. Other examples of intervals are the set of all real numbers, the set of all negative real numbers, and the empty set. Real intervals play an important role in the theory of integration, because they are the simplest sets whose "size" or "measure" or "length" is easy to define. The concept of measure can then be extended to more complicated sets of real numbers.

2.3.2. Fundamental Concepts

Our concern here is the situation where the value of a member s of a set is uncertain. We assume, however, that the information on the uncertain value of s provides an acceptable range:

$$\underline{s} \leq s \leq \overline{s} \qquad (2.6)$$

where $[\underline{s}, \overline{s}] \subset R$ is called the interval of confidence about the values of s. As a special case, we have the certainty of confidence $[\underline{s}, \overline{s}]$ = [s , s] = s. We mainly study closed intervals in this thesis; so an interval will always mean a closed and bounded interval throughout, unless otherwise indicated. In the two-dimensional case, an interval of confidence has rectangular shape as shown in Figure (2.7) and is sometimes called the region of confidence.

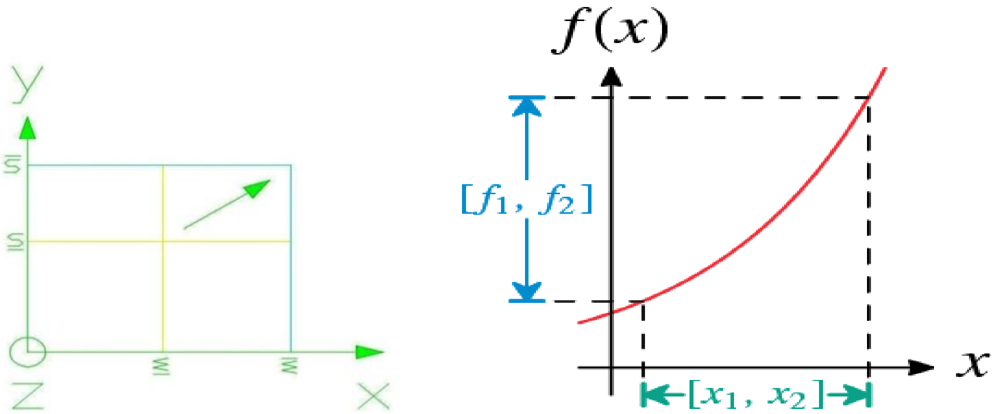

Figure (2.7) An interval of confidence in the two-dimensional case.

In the next subsection, we will introduce operational rules among intervals of confidence, which are important and useful in their own right in regards to engineering applications that are relative to intervals such as robust modeling, robust stability, and robust control.

2.4 Robust Control

2.4.1. Applications

There are a variety of complex system that required sophisticated control strategies to achieve acceptable performance within the uncertain environment in which they operate.

2.4.2. General Description

Reflects a general description of a control system as shown in Figure (2.8). The process in viewed as an element of a larger set of plants that reflects the modeling error the system is subjected to exogenous inputs such as disturbance noise or commands. These inputs represented in general the input signal uncertainty [12].

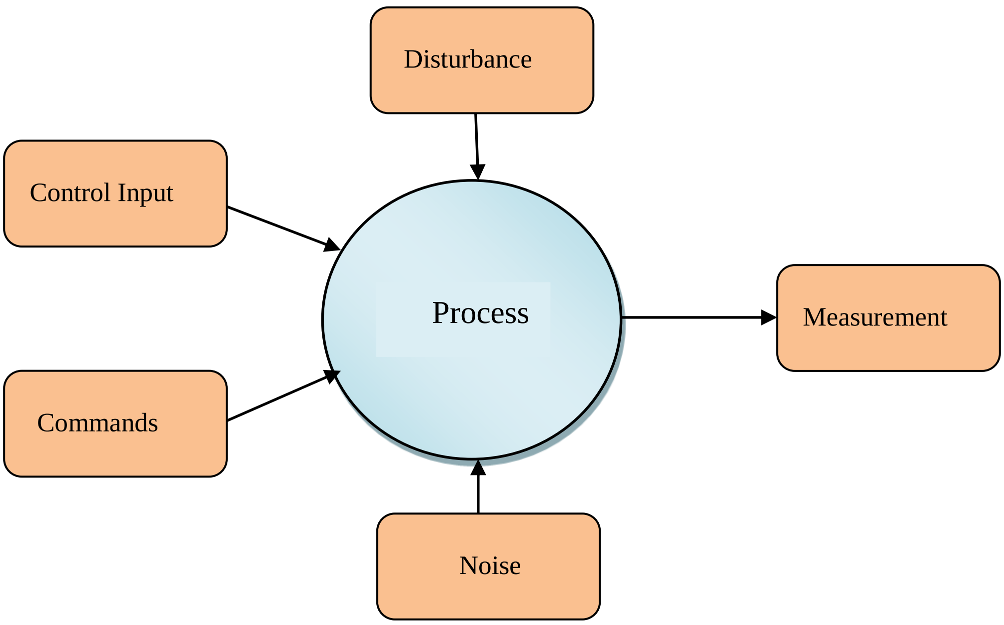

Figure (2.8) General Uncertain System

2.4.3. Stability and Performance Robustness

The major problem in controller design is finding a controller that can deliver good performance in the presence of uncertainty both in the model and the input. The following two problems:

1. **Stability Robustness Problem**
   Find a feedback controller such that a system is stable for all plants in the set of plant uncertainty.

2. **Performance Robustness problem**
   Find a feedback controller such that a system is stable and meets the desired performance objective of all plants in the set of plant uncertainty in the presence of all possible exogenous inputs.

- Modeling uncertainty are always present reasons:
  1. Incorrect parameter values.
  2. Environmental noise.
  3. Un-modeled dynamics actuators sensor.
  4. Time delays.

Modeling uncertainties are usually small at low frequency and increase as the frequency increase, therefore it is important to molded uncertainty.

- **Structured uncertainty**: The uncertainties can be described by variations of system coefficients within certain intervals around the coefficients of the nominal model .

- **Unstructured uncertainty**: is another mathematical representation of uncertainty is by a block of dynamic system set with bounded amplitude frequency response connected in series with a weighting transfer function. This weighting function represents the maximal dispersion of uncertain system frequency responses around the nominal system frequency response [12].

Chapter Three

Solution of Optimal Force Distribution Problem by Using LP

3.1 Introduction

In this chapter, we will review the linear programming defines a particular class of optimization problems in which the constraints of the system can be expressed as linear equations or inequalities and the objective function is a linear function of the design variables. Linear programming (LP) techniques are widely used to solve a number of military, economic, industrial, and societal problems. The primary reasons for its wide use are the availability of commercial software to solve very large problems and the ease with which data variation (sensitivity analysis) can be handled through LP models.

Formulation refers to the construction of LP models of real problems. Model building is not a science; it is primarily an art that is developed mainly by experience. The basic steps involved in formulating an LP model are to identify the design/decision variables, express the constraints of the problem as linear equations or inequalities, and write the objective function to be maximized or minimized as a linear function. We shall illustrate the basic steps in formulation [19].

There are two ways to solve the liner programming:

1. The Simplex method.
2. The Interior Point Methods for Linear Programming " Karmarkar's algorithm".

So we will used the first method in distribution in multi-finger dexterous hands for robotic systems because the simplex method is easier and need less computation process than Karmarkar's method.

3.2 Problem Statement

The problem of finding the optimal force distribution of an m-finger dexterous hand is to find the contact forces C_i for $1 \leq i \leq m$ that optimize a performance index subject to the force balance constraint in Eq. (2.1) and friction-force constraints in one of Eqs.(2.2)–(2.5).

A typical performance measure in this case is the weighted sum of the m normal force components c_{i1} $(1 \leq i \leq m)$, so:

$$p = \sum_{i=1}^{m} w_i C_{i1} \qquad (3.1)$$

If we employ the point-contact model and let

$$C = \begin{bmatrix} C_1 \\ \vdots \\ C_m \end{bmatrix}, \qquad C_i = \begin{bmatrix} C_{i1} \\ C_{i2} \\ C_{i3} \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}, \quad w_i = \begin{bmatrix} w_i \\ 0 \\ 0 \end{bmatrix}$$

Then the objective function in Eq. (3.1) can be expressed as

$$p(C) = w^T C \tag{3.2}$$

And the friction-force constraints in Eq. (2.3) can be written as

$$Ac \geq 0 \tag{3.3}$$

where

$$A = \begin{bmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_m \end{bmatrix} \qquad and \qquad A_i = \begin{bmatrix} 1 & 0 & 0 \\ \mu_i/\sqrt{2} & -1 & 0 \\ \mu_i/\sqrt{2} & 1 & 0 \\ \mu_i/\sqrt{2} & 0 & -1 \\ \mu_i/\sqrt{2} & 0 & 1 \end{bmatrix}$$

Obviously, the problem of minimizing function p(c) in Eq. (3.2) subject to the linear inequality constraints in Eq. (3.3) and linear equality constraints [1].

$$Wc = -f_{ext} \tag{3.4}$$

Where:

f_{ext}: External Force .
C: Contact Force.
W: The direction of the m contact forces.

Example (1) : Find the optimal contact forces C_i for $i = 1, 2, \ldots, 4$, that minimize the objective function in Eq.(3.2) subject to the constraints in Eq. (3.3) and (3.4) for a four-finger robot hand grasping the rectangular object illustrated in Figure (3.1) [9].

Solution The input data of the problem is given as follows:

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -a_1 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & a_1 & 0 \\ 0 & 1 & 0 & 0 & 0 & a_2 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & -a_2 & 0 \\ 0 & -1 & 0 & 0 & 0 & -a_3 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & -a_3 & 0 \\ 0 & -1 & 0 & 0 & 0 & a_4 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & a_4 & 0 \end{bmatrix} \tag{3.5}$$

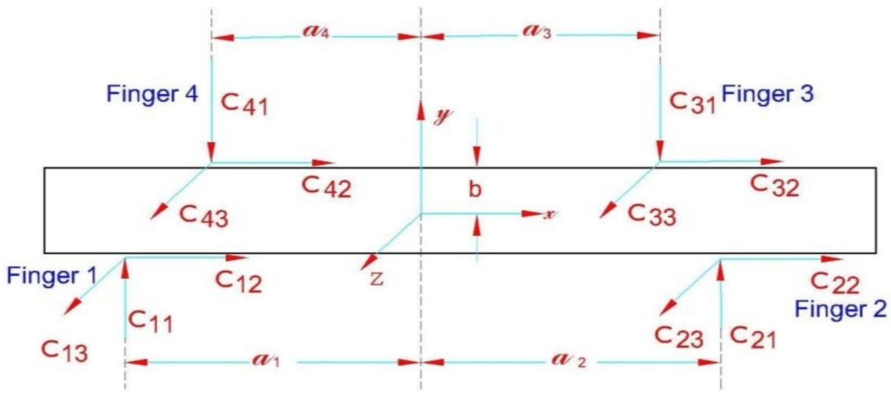

Figure (3.1) Grasping a rectangular object with four fingers.

Where $a_1 = 0.1$, $a_2 = 0.15$, $a_3 = 0.05$, $a_4 = 0.065$, and $b = 0.02$. The weights, μ_i, and f_{ext} are given by $w_i = 1$, $\mu_i = 0.4$ for $1 \le i \le 4$, and

$$f_{ext} = [0 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0]^T \tag{3.6}$$

The rank of matrix W is 6 ; hence the solutions of Eq. (3.5) can be characterized by the equation

$$C = -W^+ f_{ext} + V_\eta \phi \tag{3.7}$$

Where W^+ denotes the Moore-Penrose pseudo inverse of W, V_η is the matrix formed using the last 6 columns of V obtained from the singular-value decomposition $W=U\Sigma V^T$, and $\phi \in R^{6\times1}$ is the free parameter vector, using Eq. (3.7), the above LP problem is reduced to [9].

$$W = U\Sigma V^T$$

Let

$$V = [V1 \quad V2]$$

$$V_1 = \begin{bmatrix} -4.994 & 0.0000 & 0 & 0.5407 & 0.0000 & -0.0000 \\ -0.0001 & 0.0000 & 0.5 & -0.099 & 0.0000 & -0.0000 \\ -0.0000 & 0.4995 & 0 & -0.0000 & -0.5475 & 0.6028 \\ -0.5006 & 0.0000 & 0 & -0.7013 & 0.0000 & -0.0000 \\ -0.0001 & -0.0000 & 0.5 & -0.0994 & -0.0000 & 0.0000 \\ 0.0000 & 0.5006 & 0 & 0.0000 & 0.7182 & 0.3824 \\ 0.5002 & 0.0000 & 0 & 0.2045 & 0.0000 & -0.0000 \\ 0.0001 & -0.0000 & 0.5 & 0.0994 & -0.0000 & 0.0000 \\ 0.0000 & 0.5002 & 0 & 0.0000 & 0.2049 & -0.5433 \\ 0.49 & 0.0000 & 0 & -0.3668 & 0.0000 & -0.0000 \\ 0.0001 & -0.000 & 0.5 & 0.0994 & -0.0000 & 0.0000 \\ -0.0000 & 0.4997 & 0 & -0.0000 & -0.3774 & -0.4419 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -0.0310 & 0.3549 & 0.1466 & 0.1634 & 0.3549 & 0.3963 \\ -0.4560 & 0.1610 & 0.0289 & -0.6831 & 0.1610 & 0.0806 \\ 0.0000 & 0.0000 & 0.2772 & -0.0000 & 0.0000 & -0.1025 \\ 0.4118 & 0.2008 & 0.0103 & -0.0805 & 0.2008 & 0.0277 \\ 0.1628 & -0.3514 & 0.2279 & 0.1870 & -0.3514 & 0.6162 \\ -0.0000 & -0.0000 & -0.2772 & 0.0000 & -0.0000 & 0.1025 \\ 0.6740 & 0.2011 & 0.0642 & -0.3722 & 0.2011 & 0.1736 \\ 0.1514 & 0.5952 & -0.1289 & 0.2471 & -0.4048 & -0.3484 \\ 0.0000 & 0.0000 & 0.6025 & -0.0000 & 0.0000 & 0.2228 \\ -0.2932 & 0.3546 & 0.0926 & 0.4550 & 0.3546 & 0.2504 \\ 0.1514 & -0.4048 & -0.1289 & 0.2471 & 0.5952 & -0.3484 \\ -0.0000 & -0.0000 & -0.6025 & 0.0000 & -0.0000 & 0.2228 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 1.0000 & 0 & 0 & 0 \\ -1.0000 & 0 & 0 & 0.0088 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.0088 & 0.0003 \\ 0 & -0.000 & 0 & 0 & -0.0348 & -0.994 \\ 0 & -0.0088 & 0 & 0 & -0.994 & 0.348 \\ -0.0088 & -0.000 & 0 & -1 & 0 & -0.000 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 2.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2013 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1974 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0394 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W^{+} = W^{T}(WW^{T})^{-1} = V \begin{bmatrix} S^{-1} \\ 0 \end{bmatrix} U^{T} \qquad (3.8)$$

$$W^{+} = \begin{bmatrix} 0 & 0.273 & 0 & 0 & 0 & -2.68 \\ 0.25 & -0.00431 & 0 & 0 & 0 & 0.493 \\ 0 & 0 & 0.278 & -15.18 & 3.3 & 0 \\ 0 & 0.2194 & 0 & 0 & 0 & 3.48 \\ 0.25 & -0.00431 & 0 & 0 & 0 & 0.493 \\ 0 & 0 & 0.221 & -9.81 & -3.3 & 0 \\ 0 & -0.241 & 0 & 0 & 0 & -0.018 \\ 0.25 & 0.0043 & 0 & 0 & 0 & -0.493 \\ 0 & 0 & 0.236 & 13.73 & -1.51 & 0 \\ 0 & -0.265 & 0 & 0 & 0 & 1.82 \\ 0.25 & 0.0043 & 0 & 0 & 0 & -0.493 \\ 0 & 0 & 0.263 & 11.26 & 1.51 & 0 \end{bmatrix} \qquad (3.9)$$

$$minimize \quad \widehat{w}^{T}\phi \qquad (3.10a)$$

$$subject\ to: \quad \hat{A}\phi \geq \hat{b} \qquad (3.10b)$$

Where:

$$\widehat{w} = V_{\eta}^{T}w\,, \qquad \hat{A} = AV_{\eta}\,, \qquad \hat{b} = AW^{+}f_{ext}$$

The reduced LP problem was solved by using Primary Dual Path following algorithm – see the Appendix, this solution method is referred to as the compact LP method. If ϕ* is the minimizer of the LP problem in Eq. (3.10), then the minimizer of the original LP problem is given by :

$$C^{*} = -W^{+}f_{ext} + V_{\eta}\phi^{*} \qquad (3.11)$$

Which leads to

$$C^{*} = \begin{bmatrix} C_1^{*} \\ C_2^{*} \\ C_3^{*} \\ C_4^{*} \end{bmatrix}$$

With

$$C_1^{*} = \begin{bmatrix} 1.062736 \\ 0.010609 \\ 0.300587 \end{bmatrix}, \qquad C_2^{*} = \begin{bmatrix} 0.705031 \\ 0.015338 \\ 0.199413 \end{bmatrix}$$

$$C_3^{*} = \begin{bmatrix} 1.003685 \\ -0.038417 \\ 0.283885 \end{bmatrix}, \qquad C_4^{*} = \begin{bmatrix} 0.764082 \\ 0.012470 \\ 0.216115 \end{bmatrix}$$

The minimum value of p(c) at c* was found to be 3.535534.

3.3 Uncertainty Linear Programming

The problem of the LP can be written as follows:

$$Wc = -f_{ext}$$

$$p(C) = w^T C$$

$$Ac \geq 0$$

Where:

W: Matrix whose columns comprise the directions of the *m* contact forces.
C: Optimal Contact Force.
f_{ext}: External Force.
P(c): The Objective Function.
Ac: Friction-Force Constraints.

Our proposed method will divided into two tracks, LP and uncertainty LP as shown in Figure (3.2).

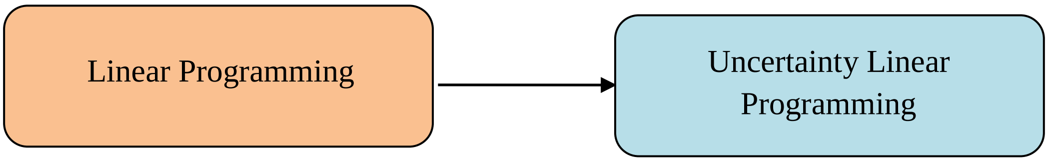

Figure (3.2) Using Uncertainty on Linear Programming system

- ☒ Uncertainty varying with one parameter:

  When $a_1 = \left[\underline{a_1}, \overline{a_1}\right]$ will solve the previous problem at two points $\underline{a_1}$and $\overline{a_1}$ the basic change will be only on W, the same change in C, so we have two points.

- ☒ Uncertainty varying with two parameter:

  When $a_1 = \left[\underline{a_1}, \overline{a_1}\right]$ will solve the previous problem at four points $\underline{a_1}$and $\overline{a_1}$, $\underline{a_2}$and $\overline{a_2}$ the basic change will be only on W as following:

  - W changes with $\underline{a_1}$and $\underline{a_2}$
  - W changes with $\underline{a_1}$and $\overline{a_2}$
  - W changes with $\overline{a_1}$and $\underline{a_2}$
  - W changes with $\overline{a_1}$and $\overline{a_2}$

The basic change will be only on W, the same change in C, so we have four points. From the LP programming equations, our variables : a_1,a_2, a_3, a_4, b, μ_i, f_{ext}, w,

A_1, A_2, A_3, A_4 so the friction force constraints $A = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ 0 & 0 & 0 & A_4 \end{bmatrix}$

And

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -a_1 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & a_1 & 0 \\ 0 & 1 & 0 & 0 & 0 & a_2 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & -a_2 & 0 \\ 0 & -1 & 0 & 0 & 0 & -a_3 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & -a_3 & 0 \\ 0 & -1 & 0 & 0 & 0 & a_4 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & a_4 & 0 \end{bmatrix}$$

With using uncertainty system:

When $a_1 = [\underline{a_1}, \overline{a_1}] = [a_1 - \Delta\,,\ a_1 + \Delta]$

When $a_2 = [\underline{a_2}, \overline{a_2}] = [a_2 - \Delta\,,\ a_2 + \Delta]$

When $a_3 = [\underline{a_3}, \overline{a_3}] = [a_3 - \Delta\,,\ a_3 + \Delta]$

When $a_4 = [\underline{a_4}, \overline{a_4}] = [a_4 - \Delta\,,\ a_4 + \Delta]$

Where $\Delta = \ a_i * \sigma$ and σ is the uncertainty value where $0 \leq \sigma \leq 1$

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -[\underline{a_1}, \overline{a_1}] \\ 1 & 0 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 1 & -0.02 & [\underline{a_1}, \overline{a_1}] & 0 \\ 0 & 1 & 0 & 0 & 0 & [\underline{a_2}, \overline{a_2}] \\ 1 & 0 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 1 & -0.02 & -[\underline{a_2}, \overline{a_2}] & 0 \\ 0 & -1 & 0 & 0 & 0 & -[\underline{a_3}, \overline{a_3}] \\ 1 & 0 & 0 & 0 & 0 & -0.02 \\ 0 & 0 & 1 & 0.02 & -[\underline{a_3}, \overline{a_3}] & 0 \\ 0 & -1 & 0 & 0 & 0 & [\underline{a_4}, \overline{a_4}] \\ 1 & 0 & 0 & 0 & 0 & -0.02 \\ 0 & 0 & 1 & 0.02 & [\underline{a_4}, \overline{a_4}] & 0 \end{bmatrix}$$

$$C_1^* = \begin{bmatrix} 1.062736 \\ 0.010609 \\ 0.300587 \end{bmatrix}, \qquad C_2^* = \begin{bmatrix} 0.705031 \\ 0.015338 \\ 0.199413 \end{bmatrix}$$

$$C_3^* = \begin{bmatrix} 1.003685 \\ -0.038417 \\ 0.283885 \end{bmatrix}, \qquad C_4^* = \begin{bmatrix} 0.764082 \\ 0.012470 \\ 0.216115 \end{bmatrix}$$

Table (3.1) LP – Uncertainty when (Δ_1=0.1, Δ_2=0.1, Δ_3=0.1, Δ_4=0.1)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
1	C_1	$C_1^* = \begin{bmatrix}1.062736\\0.010609\\0.300587\end{bmatrix}$	Δ_1 =0.1	0.9741	1.1438
	C_2	$C_2^* = \begin{bmatrix}0.705031\\0.015338\\0.199413\end{bmatrix}$	Δ_2 =0.1	0.6239	0.7937
	C_3	$C_3^* = \begin{bmatrix}1.003685\\-0.03842\\0.300587\end{bmatrix}$	Δ_3 =0.1	0.9111	1.0849
	C_4	$C_4^* = \begin{bmatrix}0.764082\\0.012470\\0.216115\end{bmatrix}$	Δ_4 =0.1	0.6828	0.8566

We can see in Figure (3.3) ULP when Δ_1=0.1 we find the first force $C_1\ lower\ bound = 0.9741\ and\ upper\ bound\ = 1.1438$, when Δ_2=0.1 we find the second force $C_2\ lower\ bound = 0.6239\ and\ upper\ bound = 0.7937$, when Δ_3=0.1 we find the third force $C_3\ lower\ bound = 0.9111\ and\ upper\ bound = 1.0849$, when Δ_4=0.1 we find the fourth force $C_4\ lower\ bound = 0.6828\ and\ upper\ bound = 0.8566$ as shown below in this figure:

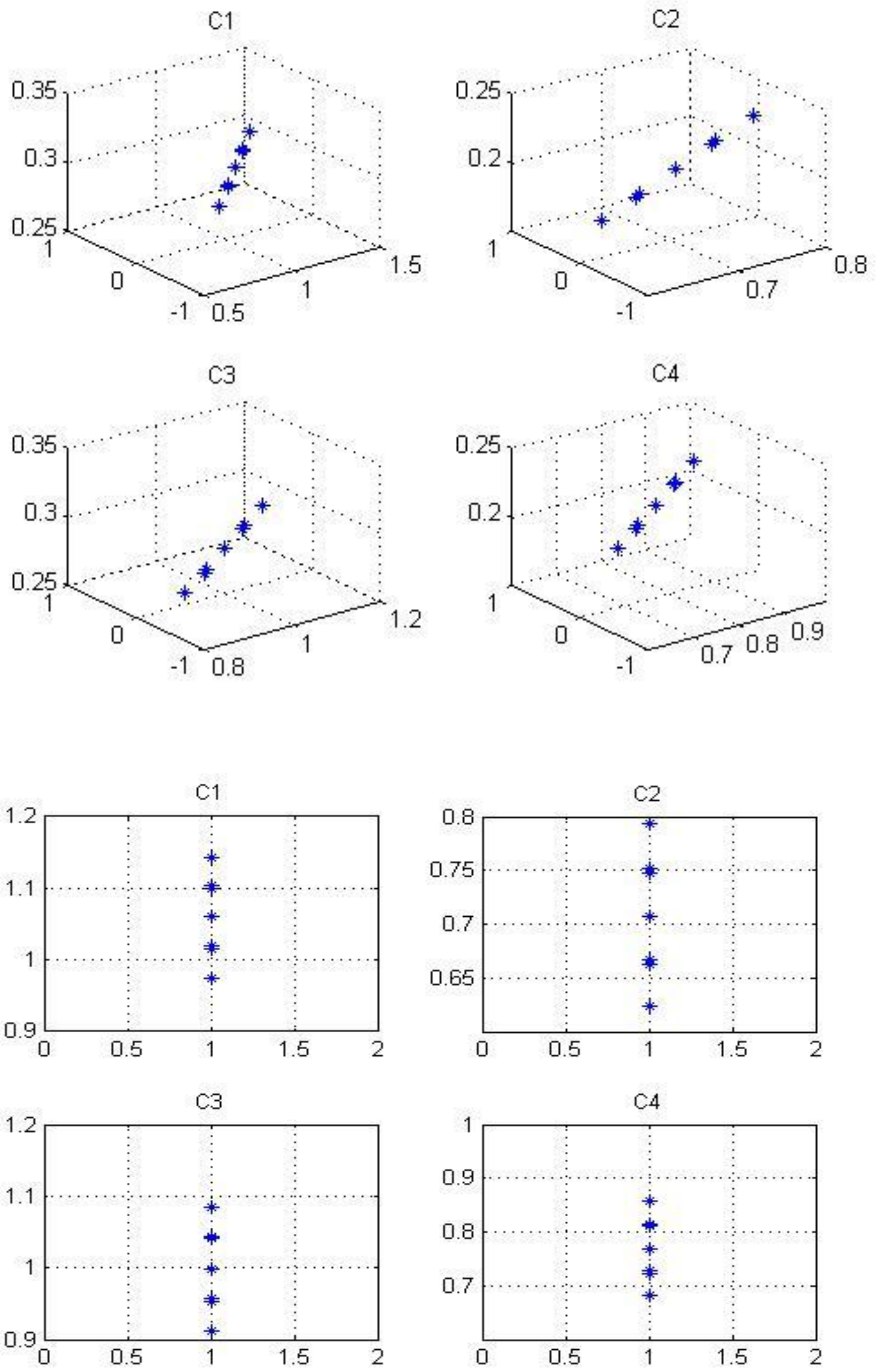

Figure (3.3) LP – Uncertainty when (Δ_1=0.1, Δ_2=0.1, Δ_3=0.1, Δ_4=0.1)

Table (3.2) LP – Uncertainty when (Δ_1=0.2, Δ_2=0.3, Δ_3=0.4, Δ_4=0.5)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
2	C1	$C_1^* = \begin{bmatrix} 1.062736 \\ 0.010609 \\ 0.300587 \end{bmatrix}$	Δ_1=0.2	0.8250	1.2535
	C2	$C_2^* = \begin{bmatrix} 0.705031 \\ 0.015338 \\ 0.199413 \end{bmatrix}$	Δ_2=0.3	0.5143	0.9428
	C3	$C_3^* = \begin{bmatrix} 1.003685 \\ -0.03842 \\ 0.300587 \end{bmatrix}$	Δ_3=0.4	0.5605	1.3518
	C4	$C_4^* = \begin{bmatrix} 0.764082 \\ 0.012470 \\ 0.216115 \end{bmatrix}$	Δ_4=0.5	0.4159	1.2073

We can see in Figure (3.4) ULP when Δ_1=0.2 we can find the first force C_1 $lower\ bound = 0.8250\ and\ upper\ bound = 1.2535$, when Δ_2 =0.3 we find the second force C_2 $lower\ bound = 0.5143\ and\ upper\ bound = 0.9428$, when Δ_3 =0.4 we find the third force C_3 $lower\ bound = 0.5605\ and\ upper\ bound = 1.3518$, when Δ_4 =0.5 we find the fourth force C_4 $lower\ bound = 0.4159\ and\ upper\ bound = 1.2073$ as shown below in this figure:

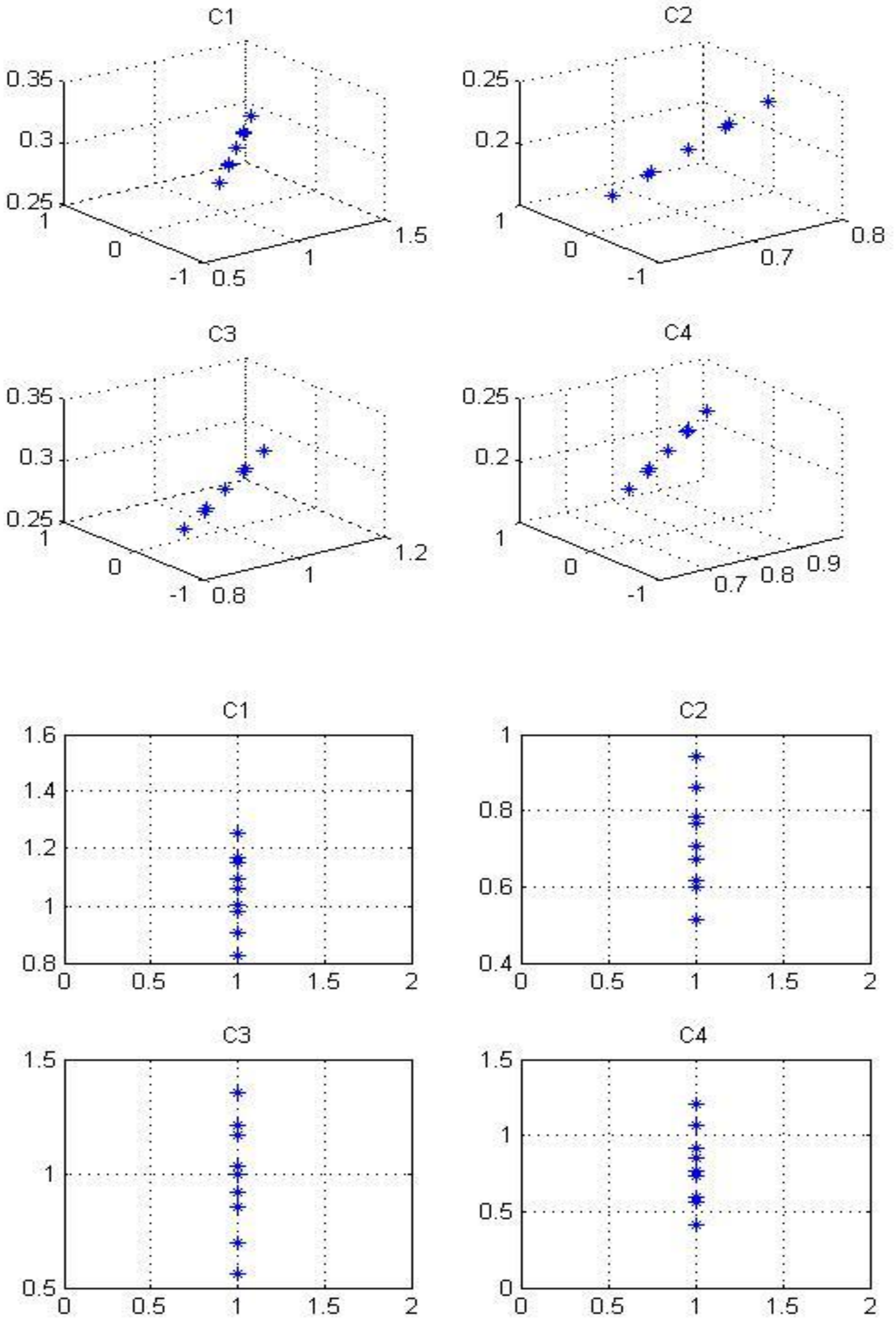

Figure (3.4) LP – Uncertainty when (Δ_1=0.2, Δ_2=0.3, Δ_3=0.4, Δ_4=0.5)

Table (3.3) LP – Uncertainty when (Δ_1=0.601, Δ_2=0.136, Δ_3=0.92, Δ_4=0.805)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
	C_1	$C_1^* = \begin{bmatrix} 1.062736 \\ 0.010609 \\ 0.300587 \end{bmatrix}$	Δ_1=0.601	0.7908	1.4324
	C_2	$C_2^* = \begin{bmatrix} 0.705031 \\ 0.015338 \\ 0.199413 \end{bmatrix}$	Δ_2=0.136	0.3354	0.9769
3	C_3	$C_3^* = \begin{bmatrix} 1.003685 \\ -0.03842 \\ 0.300587 \end{bmatrix}$	Δ_3=0.920	0.2090	1.7094
	C_4	$C_4^* = \begin{bmatrix} 0.764082 \\ 0.012470 \\ 0.216115 \end{bmatrix}$	Δ_4=0.805	0.0584	1.5588

We can see in Figure (3.5) ULP when Δ_1=0.601 we can find the first force C_1 *lower bound* = 0.7908 *and upper bound* = 1.4324, when Δ_2=0.136 we find the second force C_2 *lower bound* = 0.3354 *and upper bound* = 0.9769, when Δ_3=0.92 we find the third force C_3 *lower bound* = 0.209 *and upper bound* = 1.7094 , when Δ_4=0.805 we find the fourth force C_4 *lower bound* = 0.0584 *and upper bound* = 1.5588 as shown below in this figure:

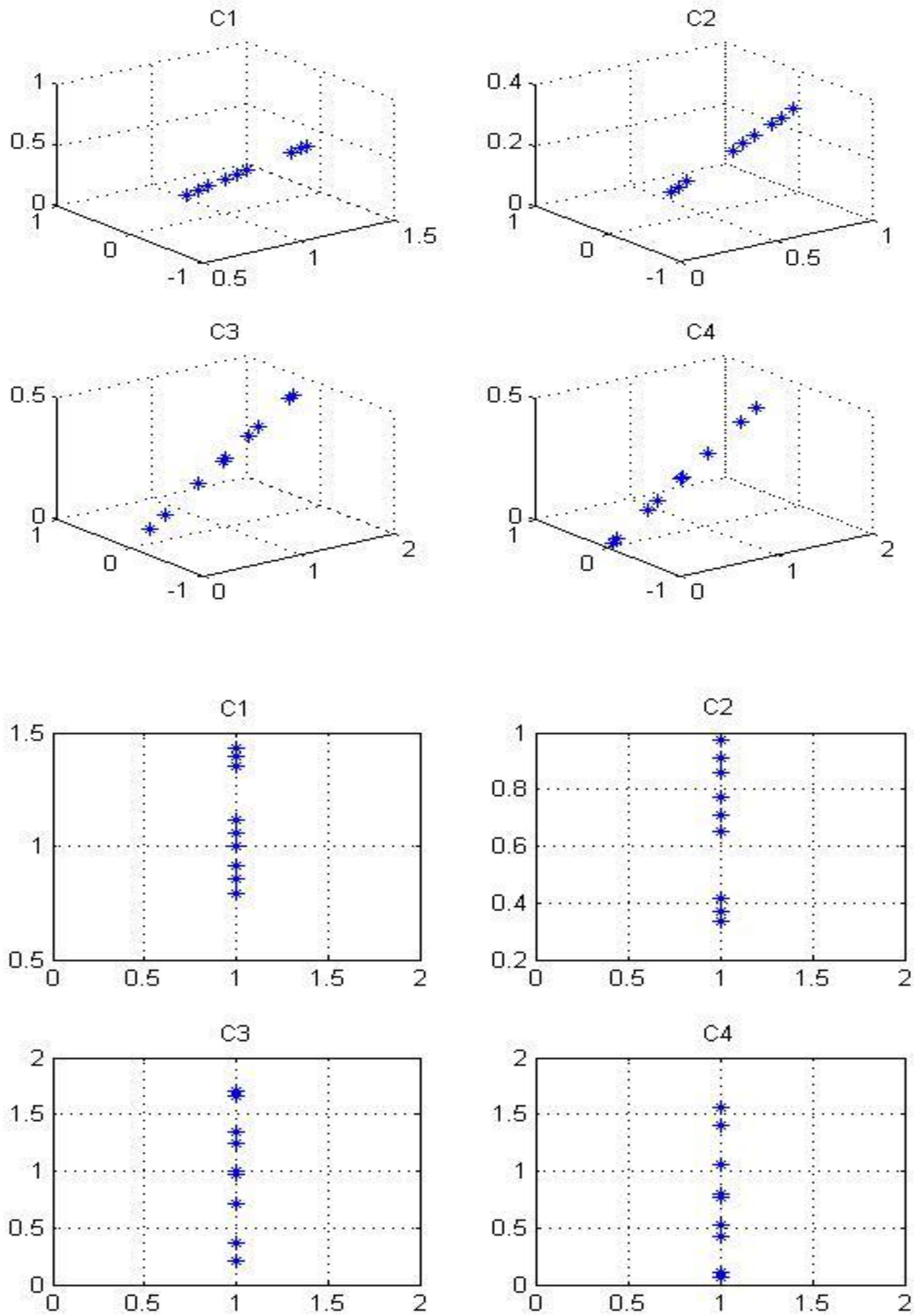

Figure (3.5) LP – Uncertainty when (Δ_1=0.601, Δ_2=0.136, Δ_3=0.92, Δ_4=0.805)

Table (3.4) LP – Uncertainty when (Δ_1=0.79, Δ_2=0.225, Δ_3=0.363, Δ_4=0.698)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
4	C_1	$C_1^* = \begin{bmatrix} 1.062736 \\ 0.010609 \\ 0.300587 \end{bmatrix}$	Δ_1=0.798	0.6960	1.5865
	C_2	$C_2^* = \begin{bmatrix} 0.705031 \\ 0.015338 \\ 0.199413 \end{bmatrix}$	Δ_2=0.225	0.1813	1.0717
	C_3	$C_3^* = \begin{bmatrix} 1.003685 \\ -0.03842 \\ 0.300587 \end{bmatrix}$	Δ_3=0.363	0.3953	1.3719
	C_4	$C_4^* = \begin{bmatrix} 0.764082 \\ 0.012470 \\ 0.216115 \end{bmatrix}$	Δ_4=0.698	0.3959	1.3724

We can see in Figure (3.6) ULP when Δ_1=0.798 we find the first force C_1 $lower\ bound = 0.696$ $and\ upper\ bound = 1.5865$, when Δ_2=0.225 we find the second force C_2 $lower\ bound = 0.1813$ $and\ upper\ bound = 1.0717$, when Δ_3=0.363 we find the third force C_3 $lower\ bound = 0.3953$ $and\ upper\ bound = 1.3719$ when Δ_4=0.698 we find the fourth force C_4 $lower\ bound = 0.3959$ $and\ upper\ bound = 1.3724$ as shown below in this figure:

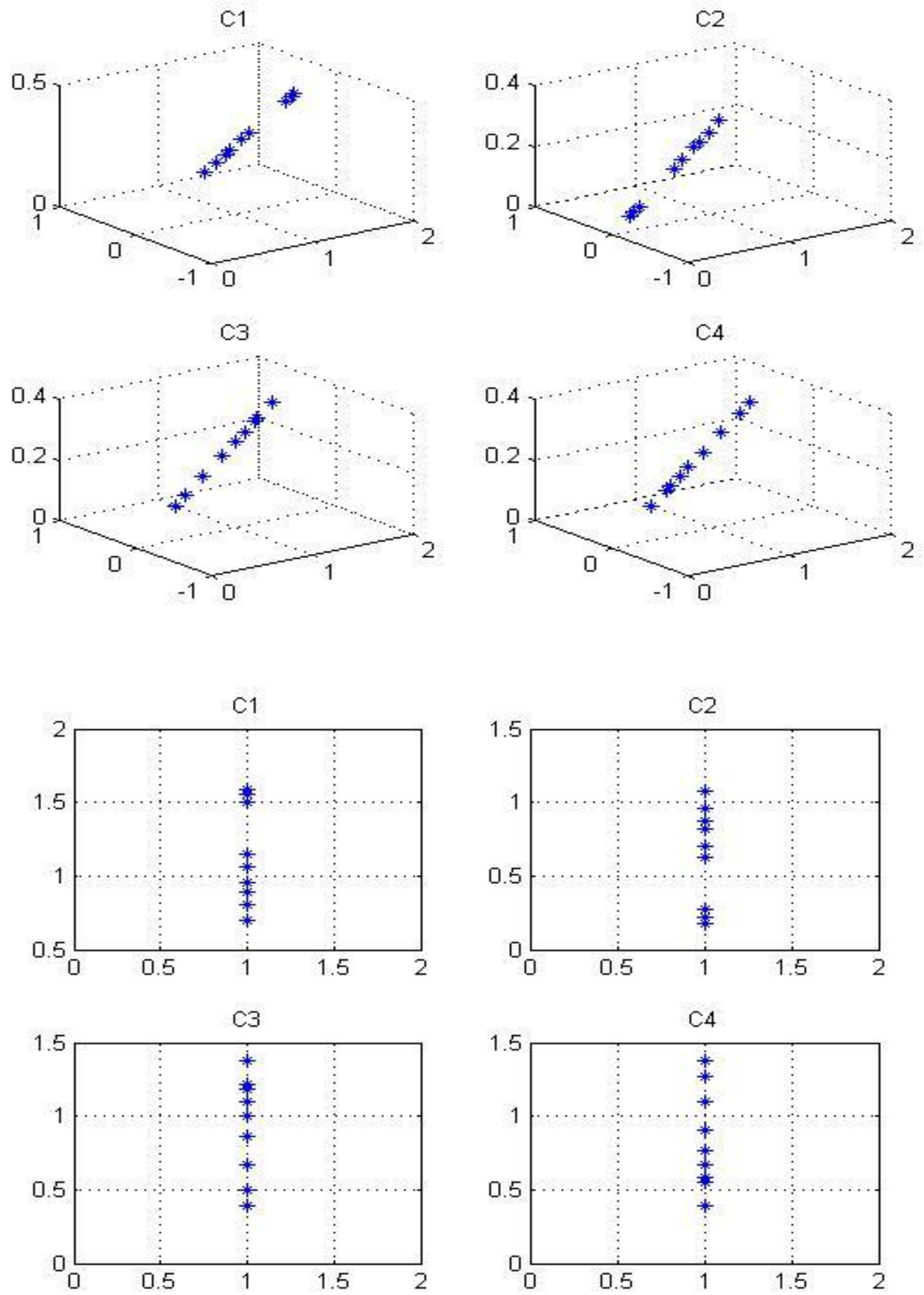

Figure (3.6) LP – Uncertainty when (Δ_1=0.79, Δ_2=0.225, Δ_3=0.363, Δ_4=0.698)

We selected a random rate uncertainty values Δ for all the values that have been applied by the ULP and we see its acceptable at this time.

Chapter Four

Solution of Optimal Force Distribution Problem by Using SDP

4.1 Introduction

There are many developed in Semidefinite programming (SDP) and it is a subfield of convex optimization concerned with the optimization of a linear objective function over the intersection of the cone of positive Semidefinite matrices with an affine space.

SDP one minimizes a linear function subject to the constraint that an affine combination of symmetric matrices is positive Semidefinite. Such a constraint is nonlinear and nonsmooth, but convex, so SDP are convex optimization problems. SDP unifies several standard problems (e.g., linear and quadratic programming) and finds many applications in engineering and combinatorial optimization.

Although SDP are much more general than linear programs, they are not much harder to solve. Most interior-point methods for linear programming have been generalized to Semidefinite programs. As in linear programming, these methods have polynomial worst-case complexity, and perform very well in practice. In addition, SDP gives a survey of the theory and applications of Semidefinite programs, and an introduction to primal-dual path methods for their solution.

In this chapter, we will review SDP is a relatively new field of optimization which is of growing interest for several reasons. Many practical problems in operations research and combinatorial optimization can be modeled or approximated as Semidefinite programming problems. In automatic control theory, SDP are used in the context of linear matrix inequalities. SDP are in fact a special case of cone programming and can be efficiently solved by interior point methods. All linear programs can be expressed as SDP, and via hierarchies of SDP the solutions of polynomial optimization problems can be approximated. Finally, Semidefinite programming has been used in the optimization of complex systems.

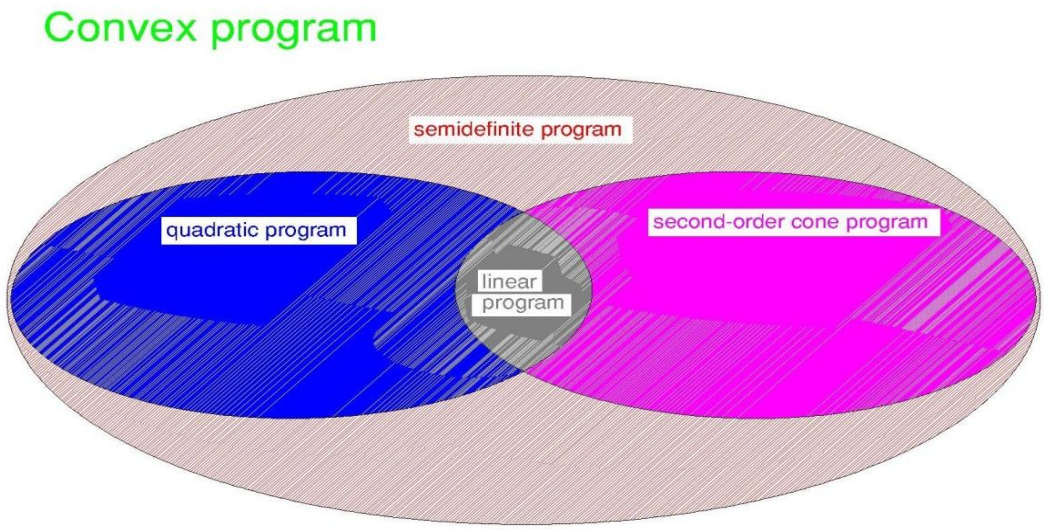

Figure (4.1) : Venn diagram of programming hierarchy.

4.2 Problem Statement

The LP-based solution discussed in solution of optimal force distribution problem by using LP is an approximate solution because it was obtained for the case where the quadratic friction-force constraint in Eq. (2.2) is approximated using a linear model. An improved solution can be obtained by formulating the problem at hand as an SDP problem. To this end, we need to convert the friction-force constraints into linear matrix inequalities.

For the point-contact case, the friction-force constraint in Eq. (2.2) yields

$$\mu_i C_{i1} \geq 0$$

And

$$\mu_i^2 C_{i1}^2 - (C_{i2}^2 + C_{i3}^2) \geq 0$$

Hence Eq. (2.2) is equivalent to

$$P_i = \begin{bmatrix} \mu_i C_{i1} & 0 & C_{i2} \\ 0 & \mu_i C_{i1} & C_{i3} \\ C_{i2} & C_{i3} & \mu_i C_{i1} \end{bmatrix} \geq 0 \quad (4.1)$$

Matrix Pi in Eq. (4.1) is positive Semidefinite if and only if:

$$\mu_i C_{i1} \geq 0\,, \quad \mu_i^2 C_{i1}^2 - C_{i2}^2 \geq 0, \quad \mu_i^2 C_{i1}^2 - C_{i3}^2 \geq 0$$

And

$$\mu_i C_{i1}(\mu_i^2 C_{i1}^2 - C_{i2}^2 - C_{i3}^2) \geq 0$$

Since the first and fourth inequalities in the above equations imply that

$$\mu_i^2 C_{i1}^2 - C_{i2}^2 - C_{i3}^2 \geq 0$$

Which, in turn, implies the second and third inequalities, the set of inequalities in can be reduced to [9].

$$\mu_i C_{i1} \geq 0 \quad \& \quad \mu_i^2 C_{i1}^2 - C_{i2}^2 - C_{i3}^2 \geq 0$$

Which is equivalent to

$$\sqrt{C_{i2}^2 + C_{i3}^2} \leq \mu_i C_{i1}$$

For an m-finger robot hand, the constraint on point-contact friction forces is given by

$$P(c) = \begin{bmatrix} P_1 & & 0 \\ & \ddots & \\ 0 & & P_m \end{bmatrix} \quad (4.2)$$

Where Pi is defined by Eq. (4.1). Similarly, the constraint on the soft-finger friction forces of an m-finger robot hand can be described by Eq. (4.2) where matrix Pi is given by [1].

$$P_i = \left[\begin{array}{cccc|ccc} C_{i1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_i & 0 & C_{i2} & 0 & 0 & 0 \\ 0 & 0 & \alpha_i & C_{i3} & 0 & 0 & 0 \\ 0 & C_{i2} & C_{i3} & \alpha_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_i & 0 & C_{i2} \\ 0 & 0 & 0 & 0 & 0 & \beta_i & C_{i3} \\ 0 & 0 & 0 & 0 & C_{i2} & C_{i3} & \beta_i \end{array}\right] \tag{4.3}$$

With $\alpha_i = \mu_i(C_{i1} + C_{i4}/\hat{\mu}_{ti})$ and $\beta_i = \mu_i(C_{i1} - C_{i4}/\hat{\mu}_{ti})$ for the linear model in Eq. (2.4) or

$$P_i = \begin{bmatrix} C_{i1} & 0 & 0 & \alpha_i C_{i2} \\ 0 & C_{i1} & 0 & \alpha_i C_{i3} \\ 0 & 0 & C_{i1} & \beta_i C_{i4} \\ \alpha_i C_{i2} & \alpha_i C_{i3} & \beta_i C_{i4} & C_{i1} \end{bmatrix} \tag{4.4}$$

Where:

$\hat{\mu}_{ti}$ is a constant between the torsion and shear limits.
μ_i is the tangential friction coefficient.

With $\alpha_i = 1/\sqrt{\mu_i}$ and with $\beta_i = 1/\sqrt{\hat{\mu}_{ti}}$ for the elliptical model in Eq. (2.5) Note that matrix P(c) for both point-contact and soft-finger models is linear with respect to parameters C_{i1}, C_{i2}, C_{i3}, and C_{i4}.
The optimal force distribution problem can now be formulated as

$$minimize\ \ p = w^T c \tag{4.5a}$$

$$subject\ to:\ \ Wc = -f_{ext} \tag{4.5b}$$

$$P(c) \geq 0 \tag{4.5c}$$

Where $C = [C_1^T \quad C_2^T \quad \cdots \quad C_m^T]^T$ with $C_i = [C_1 \quad C_2 \quad C_3]^T$ for the point-contact case or $C_i = [C_1 \quad C_2 \quad C_3 \quad C_4]^T$ for the soft-finger case, and P(c) is given by Eq. (4.2) with Pi defined by Eq. (4.1) for the point-contact case or Eq. (4.4) for the soft-finger case. By using the variable elimination method so the solutions of Eq. (4.5b) can be expressed as

$$C = V_\eta \phi + C_o \tag{4.6}$$

With $C_o = -W^+ f_{ext}$ where W^+ is the Moore-Penrose pseudo-inverse of W.
Thus the problem in Eq. (4.5) reduces to

$$minimize\ \ \hat{p} = \widehat{w}^T \phi \tag{4.7a}$$

$$subject\ to:\ \ P(V_\eta \phi + C_o) \geq 0 \tag{4.7b}$$

We need to find how we get 10 variables from 16 variables the optimal force distribution problem in with the additional requirement that λ_{min} of P(c) be no less than ε= 0.05.

$$minimize \quad \hat{w}^T \phi$$

$$subject\ to: \ -\varepsilon I + P(V_\eta \phi + C_0) \geq 0$$

If we let ($y = \phi$) then the two equation can be

$$maximize \quad \hat{w}^T y$$

$$subject\ to: \ -\varepsilon I + P(C_0 - V_\eta y) \geq 0$$

By using dual path so: $c = [C_1 \quad C_2 \quad C_3 \quad \cdots \quad C_{16}]^T$ for express matrix P(c) as

$$P(c) = P_0 + P_1 C_1 + \cdots\cdots\cdots + P_{16} C_{16}$$

For c_i can be written as:

$$C_i = e^T(C_o + V_\eta \phi) = C_{oi} + V_i^T \phi = C_{oi} - V_{iy}^T = C_{oi} - \sum_{j=1}^{10} V_{ij} y_j$$

So:

$$P(C_o - V_\eta y) = P_o + \sum_{i=1}^{16} C_i P_i = P_o + \sum_{i=1}^{16} \left(C_o - \sum_{j=1}^{10} V_{ij} y_j \right) P_i$$

$$P(C_o - V_\eta y) = \left(P_o + \sum_{i=1}^{16} C_{oi} P_i \right) - \sum_{j=1}^{10} y_j (\sum_{i=1}^{16} V_{ij} P_i)$$

$$P(C_o - V_\eta y) = C - \sum_{j=1}^{10} y_j A_j$$

Where:

$$C = P_o + \sum_{i=1}^{16} C_{oi} P_i$$

$$A_j = \sum_{i=1}^{16} V_{ij} P_i$$

C_{oi} : *is the* i^{th} *component of* C_0 and
V_i : *is the* i^{th} *row of* V_η *consequantly we have*

Since $P(V_\eta \phi + C_o)$ is affine with respect to vector ϕ, the optimization problem in Eq.(4.7) is a standard SDP.

Example (2): Find the optimal contact forces Ci for 1 ≤ i ≤ 4 that would solve the minimization problem in Eq. (4.5) for the 4-finger robot hand grasping the rectangular object illustrated in Figure (3.1), using the soft-finger model in Eq.(2.5) with $\mu_i = 0.4 \ and \ \mu_{ti} = \sqrt{0.2}$ for $1 \le i \le 4$ [9].

Solution The input data are given by

$$w = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$$

$$f_{ext} = [1 \quad -1 \quad 1 \quad 0 \quad 0.5 \quad 0.5]^T$$

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -a_1 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & a_1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & a_2 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & -a_2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -a_3 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & -a_3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & a_4 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & a_4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W = U \Sigma V^T$$

Let: $V=[V_1 \ V_2]$

$$V_1 = \begin{bmatrix} 0 & -0.4995 & 0 & 0 & 0.5407 & 0 & 0.248 & -0.248 \\ 0 & -0.0001 & 0.5 & 0 & -0.0994 & 0 & 0.225 & -0.225 \\ 0.208 & -0.0000 & 0 & -0.4579 & -0.0000 & 0.501 & -0.031 & 0.031 \\ 0.428 & 0.0000 & 0 & 0.2535 & 0.0000 & -0.008 & -0.752 & -0.248 \\ 0 & -0.5006 & 0 & 0 & -0.7013 & 0 & 0.125 & -0.125 \\ 0 & -0.0001 & 0.5 & 0 & -0.0994 & 0 & 0.052 & -0.052 \\ 0.315 & 0.0000 & 0 & -0.3945 & 0.0000 & 0.499 & 0.031 & -0.031 \\ 0.428 & 0.0000 & 0 & 0.2534 & 0.0000 & -0.008 & 0.248 & 0.752 \\ 0 & 0.5002 & 0 & 0 & 0.2045 & 0 & 0.254 & -0.254 \\ 0 & 0.0001 & 0.5 & 0 & 0.0994 & 0 & -0.139 & 0.139 \\ 0.272 & 0.0000 & 0 & -0.4198 & 0.0000 & -0.500 & 0.014 & -0.014 \\ -0.428 & -0.0000 & 0 & -0.2534 & -0.0000 & 0.008 & -0.248 & 0.248 \\ 0 & 0.4997 & 0 & 0 & 0.3668 & 0 & 0.118 & -0.118 \\ 0 & 0.0001 & 0.5 & 0 & 0.0994 & 0 & -0.139 & 0.139 \\ 0.223 & -0.0000 & 0 & -0.4490 & -0.0000 & -0.499 & -0.014 & 0.014 \\ -0.428 & -0.0000 & 0 & -0.2534 & -0.0000 & 0.008 & -0.248 & 0.248 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -0.078 & 0.1387 & 0.0167 & 0.2479 & 0.4069 & 0.1387 & 0.0452 & 0.2479 \\ -0.493 & 0.0345 & 0.0152 & 0.2251 & -0.5389 & 0.0345 & 0.0411 & 0.2251 \\ 0.0000 & 0.0000 & 0.4979 & -0.0310 & -0.0000 & 0.0000 & 0.4944 & -0.0310 \\ -0.0000 & -0.0000 & 0.0167 & 0.2477 & 0.0000 & -0.0000 & 0.0452 & 0.2477 \\ 0.3978 & 0.1349 & 0.0084 & 0.1248 & -0.0063 & 0.1349 & 0.0228 & 0.1248 \\ 0.1246 & -0.5304 & 0.0035 & 0.0522 & 0.3886 & -0.5304 & 0.0095 & 0.0522 \\ -0.0000 & -0.0000 & -0.4979 & 0.0310 & 0.0000 & -0.0000 & -0.4944 & 0.0310 \\ -0.0000 & -0.0000 & 0.0167 & 0.2477 & 0.0000 & -0.0000 & 0.0452 & 0.2477 \\ 0.6391 & 0.0376 & 0.0172 & 0.2444 & -0.1880 & 0.0376 & 0.0464 & 0.2544 \\ 0.1840 & 0.7479 & -0.0094 & -0.1387 & 0.0751 & -0.2521 & -0.0253 & -0.1387 \\ -0.0000 & -0.0000 & 0.5010 & 0.0142 & 0.0000 & -0.0000 & -0.4974 & 0.0142 \\ 0.0000 & 0.0000 & -0.0167 & 0.7523 & -0.0000 & 0.0000 & -0.0452 & -0.2477 \\ -0.3185 & 0.236 & 0.0080 & 0.1183 & 0.5885 & 0.2360 & 0.0216 & 0.1183 \\ 0.1840 & -0.2521 & -0.0094 & -0.1387 & 0.0751 & 0.7479 & -0.253 & -0.1387 \\ 0.0000 & 0.0000 & -0.5010 & -0.0142 & -0.0000 & 0.0000 & 0.4974 & -0.0142 \\ 0.0000 & 0.0000 & -0.0167 & -0.2477 & -0.0000 & 0.0000 & -0.0452 & 0.7523 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & 0 & 1.000 & 0 & 0 & 0 \\ 0.000 & -1.000 & 0 & -0.000 & 0.0088 & -0.000 \\ 0.5055 & 0 & 0 & -0.8628 & 0 & 0.000 \\ -0.0003 & 0 & 0 & 0.0002 & 0 & -1.000 \\ -0.8628 & 0 & 0 & -0.5055 & 0 & 0.0003 \\ -0.000 & -0.0088 & 0 & 0.000 & -1.000 & 0.000 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 2.015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.995 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2013 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.04 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W^+ = W^T(WW^T)^{-1} = V \begin{bmatrix} S^{-1} \\ 0 \end{bmatrix} U^T \qquad (4.8)$$

$$W^{+} = \begin{bmatrix} 0 & 0.2734 & 0 & 0 & 0 & -2.68 \\ 0.25 & -0.00431 & 0 & 0 & 0 & 0.493 \\ 0 & 0 & 0.25 & -12.5 & 0.030 & 0 \\ 0 & 0 & -0.0021 & 0.201 & -0.247 & 0 \\ 0 & 0.291 & 0 & 0 & 0 & 3.486 \\ 0.25 & -0.00431 & 0 & 0 & 0 & 0.493 \\ 0 & 0 & 0.249 & -12.47 & -0.030 & 0 \\ 0 & 0 & -0.0021 & 0.201 & -0.247 & 0 \\ 0 & -0.241 & 0 & 0 & 0 & -1.01 \\ 0.25 & 0.00431 & 0 & 0 & 0 & -0.493 \\ 0 & 0 & 0.249 & 12.51 & -0.014 & 0 \\ 0 & 0 & 0.0021 & -0.201 & 0.247 & 0 \\ 0 & -0.265 & 0 & 0 & 0 & 1.82 \\ 0.25 & 0.00431 & 0 & 0 & 0 & -0.493 \\ 0 & 0 & 0.25 & 12.48 & 0.0142 & 0 \\ 0 & 0 & 0.00216 & -0.201 & 0.247 & 0 \end{bmatrix}$$

Where the numerical values of a_1, a_2, a_3, a_4 and b are the same as in Example (1). By applying Primary dual path algorithm to the SDP problem in Eq. (4.7), the minimizer ϕ^* was found to be

$$\phi^* = \begin{bmatrix} -2.419912 \\ -0.217252 \\ 3.275539 \\ 0.705386 \\ -0.364026 \\ -0.324137 \\ -0.028661 \\ 0.065540 \\ -0.839180 \\ 0.217987 \end{bmatrix}$$

Eq. (4.6) then yields

$$C^* = V_\eta \phi^* + C_o = \begin{bmatrix} C_1^* \\ C_2^* \\ C_3^* \\ C_4^* \end{bmatrix}$$

Where

$$C_1^* = \begin{bmatrix} 2.706396 \\ -1.636606 \\ 0.499748 \\ -0.015208 \end{bmatrix}, \quad C_2^* = \begin{bmatrix} 0.003041 \\ -0.000633 \\ 0.000252 \\ -0.000172 \end{bmatrix}$$

$$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.541217 \end{bmatrix}, \quad C_4^* = \begin{bmatrix} 0.009955 \\ -0.001303 \\ -0.000059 \\ 0.000907 \end{bmatrix}$$

The minimum value of p(c) at c* is p(c)* = 6.418873.

4.3 Uncertainty Semidefinite Programming:

The problem of the SDP can be written as follows:

$$minimize\ \ p = w^T c$$

$$subject\ to:\ \ Wc = -f_{ext}$$

$$P(c) \geq 0$$

Where:

W: Matrix whose columns comprise the directions of the *m* contact forces.
C: Optimal Contact Force.
f_{ext}: External Force.
P(c): The Objective Function.

Our proposed method will divided into two tracks, SDP and uncertainty SDP as shown in Figure (4.1).

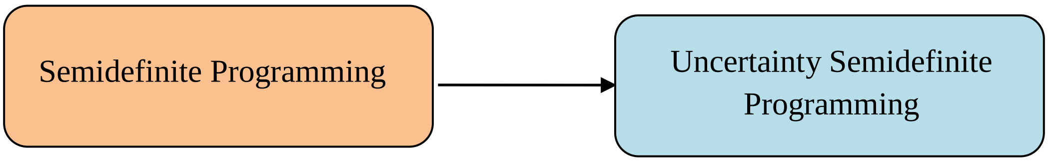

Figure (4.2) Using Uncertainty on SDP Programming system

- Uncertainty varying with one parameter:
  When $a_1 = [\underline{a_1}, \overline{a_1}]$ will solve the previous problem at two points $\underline{a_1}$ and $\overline{a_1}$ the basic change will be only on W, the same change in C, so we have two points.

- Uncertainty varying with two parameter:
  When $a_1 = [\underline{a_1}, \overline{a_1}]$ will solve the previous problem at four points $\underline{a_1}$ and $\overline{a_1}$, $\underline{a_2}$ and $\overline{a_2}$ the basic change will be only on W as following:

  - W changes with $\underline{a_1}$ and $\underline{a_2}$
  - W changes with $\underline{a_1}$ and $\overline{a_2}$
  - W changes with $\overline{a_1}$ and $\underline{a_2}$
  - W changes with $\overline{a_1}$ and $\overline{a_2}$

  The basic change will be only on W, the same change in C, so we have four points.

From the SDP programming equations, our variables : a_1,a_2, a_3, a_4, b, μ_i, f_{ext}, w,

$$w = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]^T$$

$$f_{ext} = [1 \quad -1 \quad 1 \quad 0 \quad 0.5 \quad 0.5]^T$$

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -a_1 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & a_1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & a_2 \\ 1 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 1 & -b & -a_2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -a_3 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & -a_3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & a_4 \\ 1 & 0 & 0 & 0 & 0 & -b \\ 0 & 0 & 1 & b & a_4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

With using uncertainty system :

When $a_1 = [\underline{a_1}, \overline{a_1}] = [a_1 - \Delta\,,\ a_1 + \Delta]$

When $a_2 = [\underline{a_2}, \overline{a_2}] = [a_2 - \Delta\,,\ a_2 + \Delta]$

When $a_3 = [\underline{a_3}, \overline{a_3}] = [a_3 - \Delta\,,\ a_3 + \Delta]$

When $a_4 = [\underline{a_4}, \overline{a_4}] = [a_4 - \Delta\,,\ a_4 + \Delta]$

Where $\Delta = a_i * \sigma$ and σ is the uncertainty value where $0 \leq \sigma \leq 1$

$$W^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -[\underline{a_1}, \overline{a_1}] \\ 1 & 0 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 1 & -0.02 & [\underline{a_1}, \overline{a_1}] & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & [\underline{a_2}, \overline{a_2}] \\ 1 & 0 & 0 & 0 & 0 & 0.02 \\ 0 & 0 & 1 & -0.02 & -[\underline{a_2}, \overline{a_2}] & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -[\underline{a_3}, \overline{a_3}] \\ 1 & 0 & 0 & 0 & 0 & -0.02 \\ 0 & 0 & 1 & 0.02 & -[\underline{a_3}, \overline{a_3}] & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & [\underline{a_4}, \overline{a_4}] \\ 1 & 0 & 0 & 0 & 0 & -0.02 \\ 0 & 0 & 1 & 0.02 & [\underline{a_4}, \overline{a_4}] & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C_1^* = \begin{bmatrix} 2.706396 \\ -1.636606 \\ 0.499748 \\ -0.015208 \end{bmatrix}, \quad C_2^* = \begin{bmatrix} 0.003041 \\ -0.000633 \\ 0.000252 \\ -0.000172 \end{bmatrix}$$

$$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.541217 \end{bmatrix}, \quad C_4^* = \begin{bmatrix} 0.009955 \\ -0.001303 \\ -0.000059 \\ 0.000907 \end{bmatrix}$$

Table (4.1) SDP – Uncertainty when (Δ_1=0.1, Δ_2=0.1, Δ_3=0.1, Δ_4=0.1)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
1	C_1	$C_1^* = \begin{bmatrix} 2.76396 \\ -1.6366 \\ 0.49974 \\ -0.015208 \end{bmatrix}$	Δ_1=0.10	2.4649	2.9898
	C_2	$C_2^* = \begin{bmatrix} 0.003040 \\ -0.00063 \\ 0.000252 \\ -0.00017 \end{bmatrix}$	Δ_2=0.10	0.0010	0.0054
	C_3	$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.54121 \end{bmatrix}$	Δ_3=0.10	3.4623	3.9828
	C_4	$C_4^* = \begin{bmatrix} 0.009955 \\ -0.00130 \\ -0.00059 \\ 0.000907 \end{bmatrix}$	Δ_4=0.10	0.0012	0.0140

We can see in Figure (4.3) USDP when Δ_1=0.1 we can find the first force C_1 $lower\ bound = 2.4649$ $and\ upper\ bound = 2.9898$, when Δ_2=0.1 we find the second force C_2 $lower\ bound = 0.001$ $and\ upper\ bound = 0.0054$, when Δ_3=0.1 we find the third force C_3 $lower\ bound = 3.4623$ $and\ upper\ bound = 3.9828$, when Δ_4= 0.1 we find the fourth force C_4 $lower\ bound = 0.0012$ $and\ upper\ bound = 0.014$, as shown below in this figure:

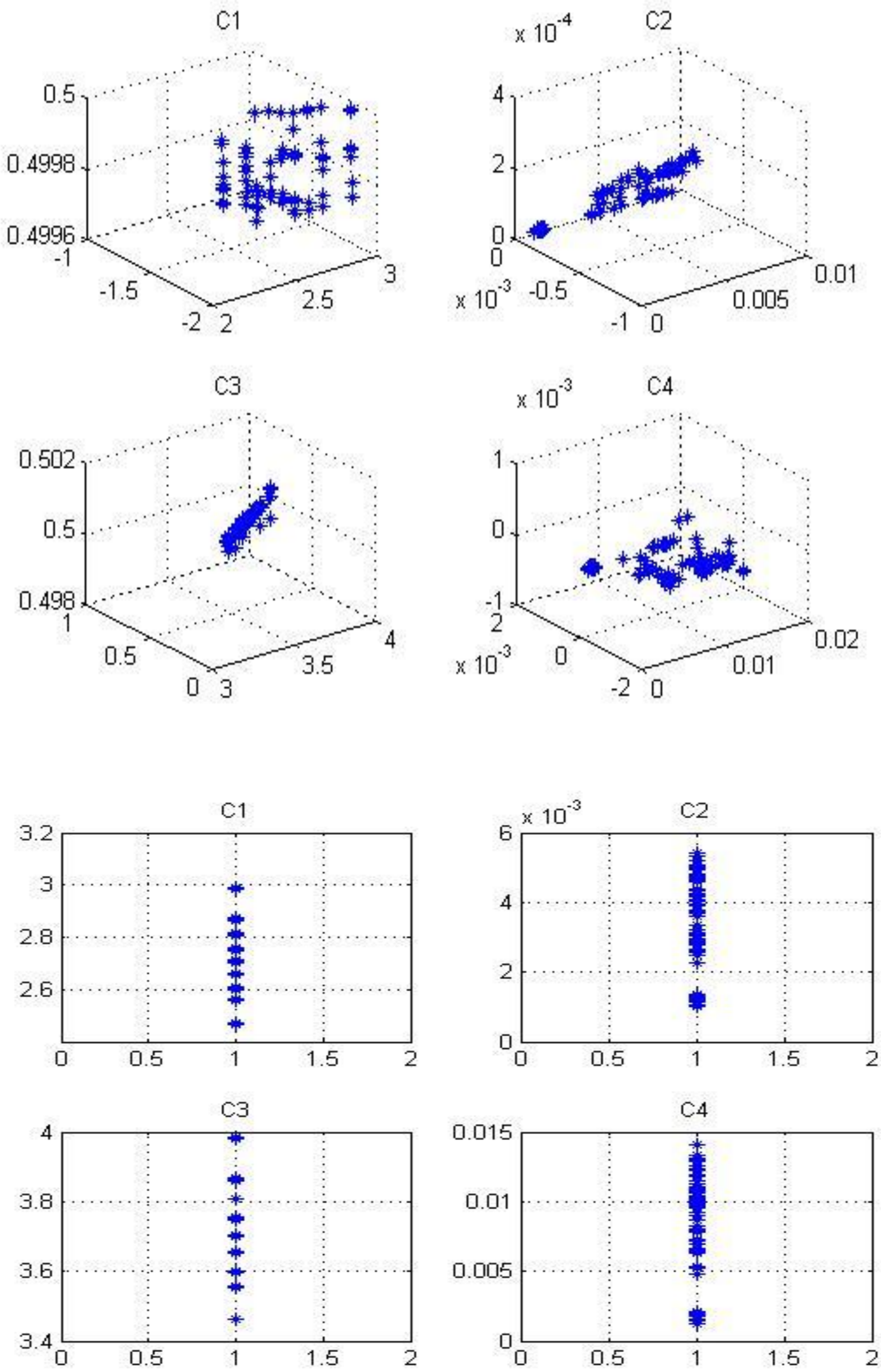

Figure (4.3) SDP –Uncertainty when (Δ_1=0.1, Δ_2=0.1, Δ_3=0.1, Δ_4=0.1)

Table (4.2) SDP –Uncertainty when (Δ_1=0.2, Δ_2=0.3, Δ_3=0.4, Δ_4=0.5)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
2	C_1	$C_1^* = \begin{bmatrix} 2.76396 \\ -1.6366 \\ 0.49974 \\ -0.015208 \end{bmatrix}$	Δ_1=0.2	2.1101	3.6529
	C_2	$C_2^* = \begin{bmatrix} 0.003040 \\ -0.00063 \\ 0.000252 \\ -0.00017 \end{bmatrix}$	Δ_2=0.3	0.0010	0.0075
	C_3	$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.54121 \end{bmatrix}$	Δ_3=0.4	3.1083	4.6428
	C_4	$C_4^* = \begin{bmatrix} 0.009955 \\ -0.00130 \\ -0.00059 \\ 0.000907 \end{bmatrix}$	Δ_4=0.5	0.0012	0.0365

We can see in Figure (4.4) USDP when Δ_1=0.2 we can find the first force C_1 $lower\ bound = 2.1101$ $and\ upper\ bound = 3.6529$, when Δ_2=0.3 we find the second force C_2 $lower\ bound = 0.001$ $and\ upper\ bound = 0.0075$, when Δ_3=0.4 we find the third force C_3 $lower\ bound = 3.1083$ $and\ upper\ bound = 4.6426$, when Δ_4= 0.5 we find the fourth force C_4 $lower\ bound = 0.0012$ $and\ upper\ bound = 0.0365$, as shown below in this figure:

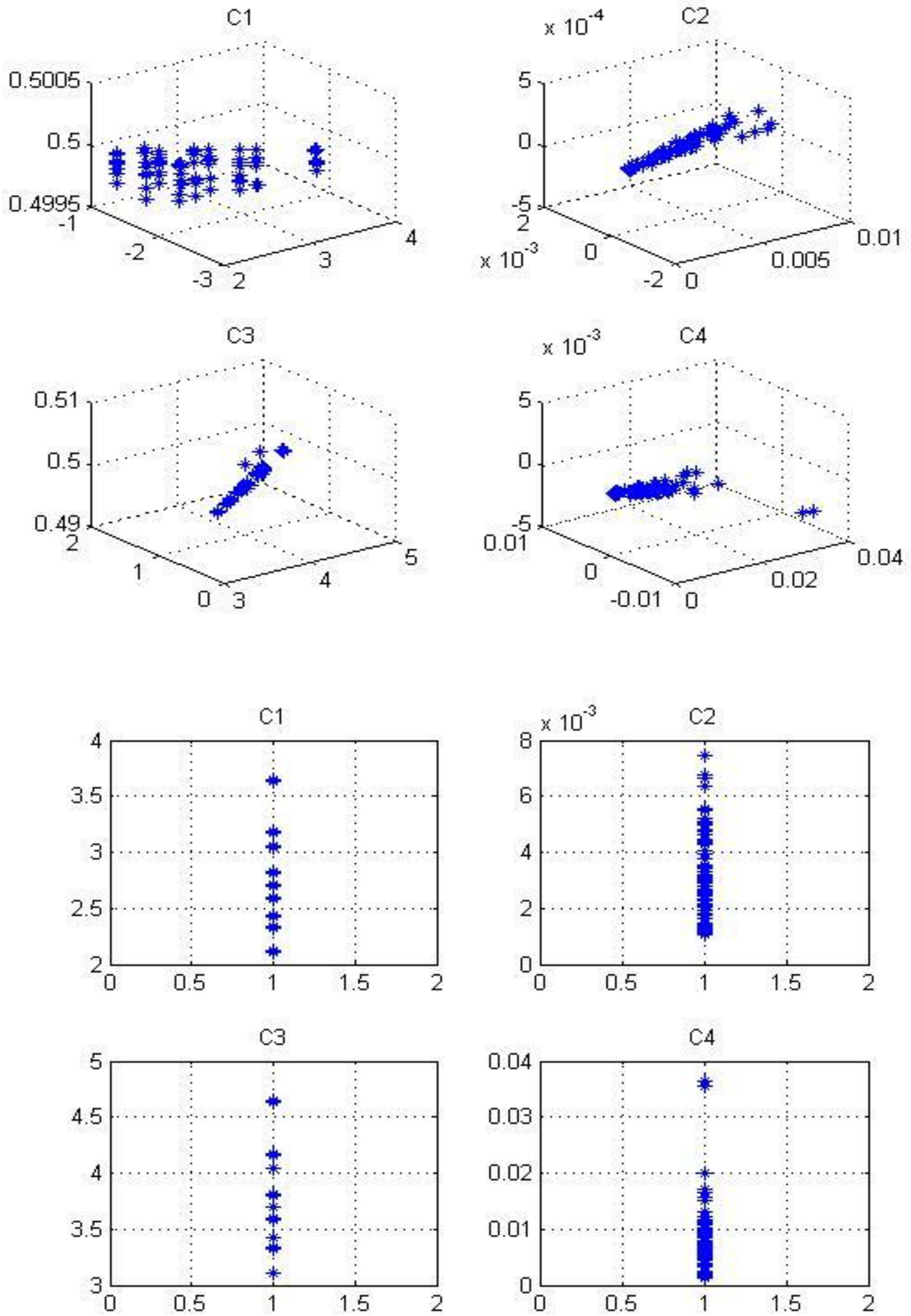

Figure (4.4) SDP –Uncertainty when (Δ_1=0.2, Δ_2=0.3, Δ_3=0.4, Δ_4=0.5)

Table (4.3) SDP –Uncertainty when (Δ_1=0.601, Δ_2=0.136, Δ_3=0.92, Δ_4=0.805)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
3	C_1	$C_1^* = \begin{bmatrix} 2.76396 \\ -1.6366 \\ 0.49974 \\ -0.015208 \end{bmatrix}$	Δ_1=0.601	1.5278	7.5072
	C_2	$C_2^* = \begin{bmatrix} 0.003040 \\ -0.00063 \\ 0.000252 \\ -0.00017 \end{bmatrix}$	Δ_2=0.136	0.0010	0.0077
	C_3	$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.54121 \end{bmatrix}$	Δ_3=0.920	2.5262	8.5014
	C_4	$C_4^* = \begin{bmatrix} 0.009955 \\ -0.00130 \\ -0.00059 \\ 0.000907 \end{bmatrix}$	Δ_4=0.805	0.0013	0.1075

We can see in Figure (4.5) USDP when Δ_1=0.601 we can find the first force C_1 $lower\ bound = 1.5278$ $and\ upper\ bound = 7.5072$, when Δ_2=0.136 we find the second force C_2 $lower\ bound = 0.001$ $and\ upper\ bound = 0.0077$, when Δ_3=0.92 we find the third force C_3 $lower\ bound = 2.5262$ $and\ upper\ bound = 8.5014$, when Δ_4= 0.805 we find the fourth force C_4 $lower\ bound = 0.0013$ $and\ upper\ bound = 0.1075$, as shown below in this figure:

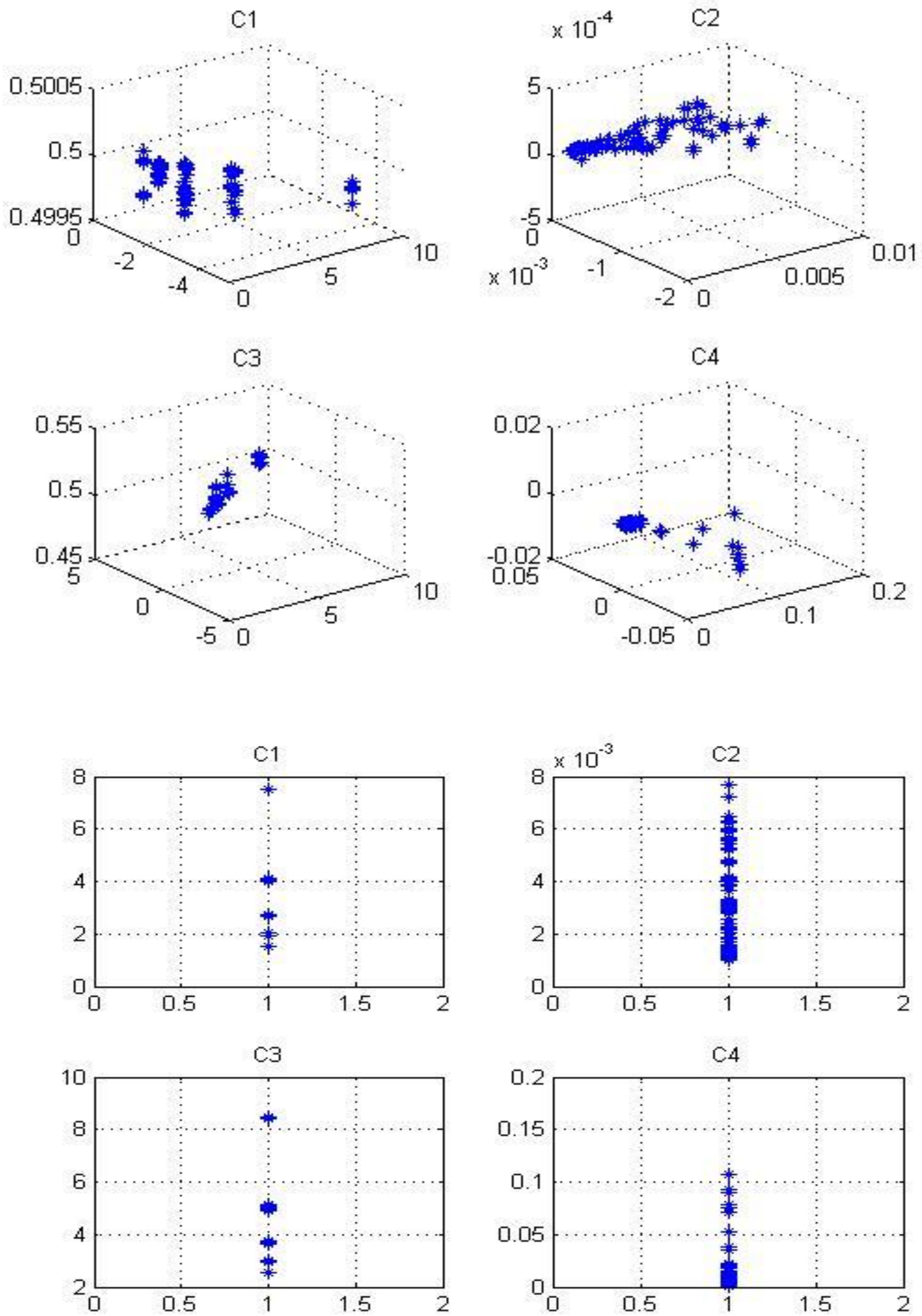

Figure (4.5) SDP –Uncertainty when (Δ_1=0.601, Δ_2=0.136, Δ_3=0.92, Δ_4=0.805)

Table (4.4) SDP – Uncertainty when (Δ1=0.79, Δ2=0.225, Δ3=0.363, Δ4=0.698)

N.	FORCE	THE MATRIX	DELTA	LOWER BOUD	UPPER BOUND
4	C_1	$C_1^* = \begin{bmatrix} 2.76396 \\ -1.6366 \\ 0.49974 \\ -0.015208 \end{bmatrix}$	Δ_1=0.798	1.6741	6.3584
	C_2	$C_2^* = \begin{bmatrix} 0.003040 \\ -0.00063 \\ 0.000252 \\ -0.00017 \end{bmatrix}$	Δ_2=0.225	0.0011	0.0085
	C_3	$C_3^* = \begin{bmatrix} 3.699481 \\ 0.638543 \\ 0.500059 \\ -0.54121 \end{bmatrix}$	Δ_3=0.363	2.6702	7.3538
	C_4	$C_4^* = \begin{bmatrix} 0.009955 \\ -0.00130 \\ -0.00059 \\ 0.000907 \end{bmatrix}$	Δ_4=0.698	0.0013	0.0241

We can see in Figure (4.6) USDP when Δ_1=0.798 we can find the first force $C_1\ lower\ bound = 1.6741\ and\ upper\ bound = 6.3584$, when Δ_2=0.225 we find the second force $C_2\ lower\ bound = 0.0011\ and\ upper\ bound = 0.0085$, when Δ_3=0.363 we find the third force $C_3\ lower\ bound = 2.6702\ and\ upper\ bound = 7.3538$, when Δ_4= 0.698 we find the fourth force $C_4\ lower\ bound = 0.0013\ and\ upper\ bound = 0.0241$, as shown below in this figure:

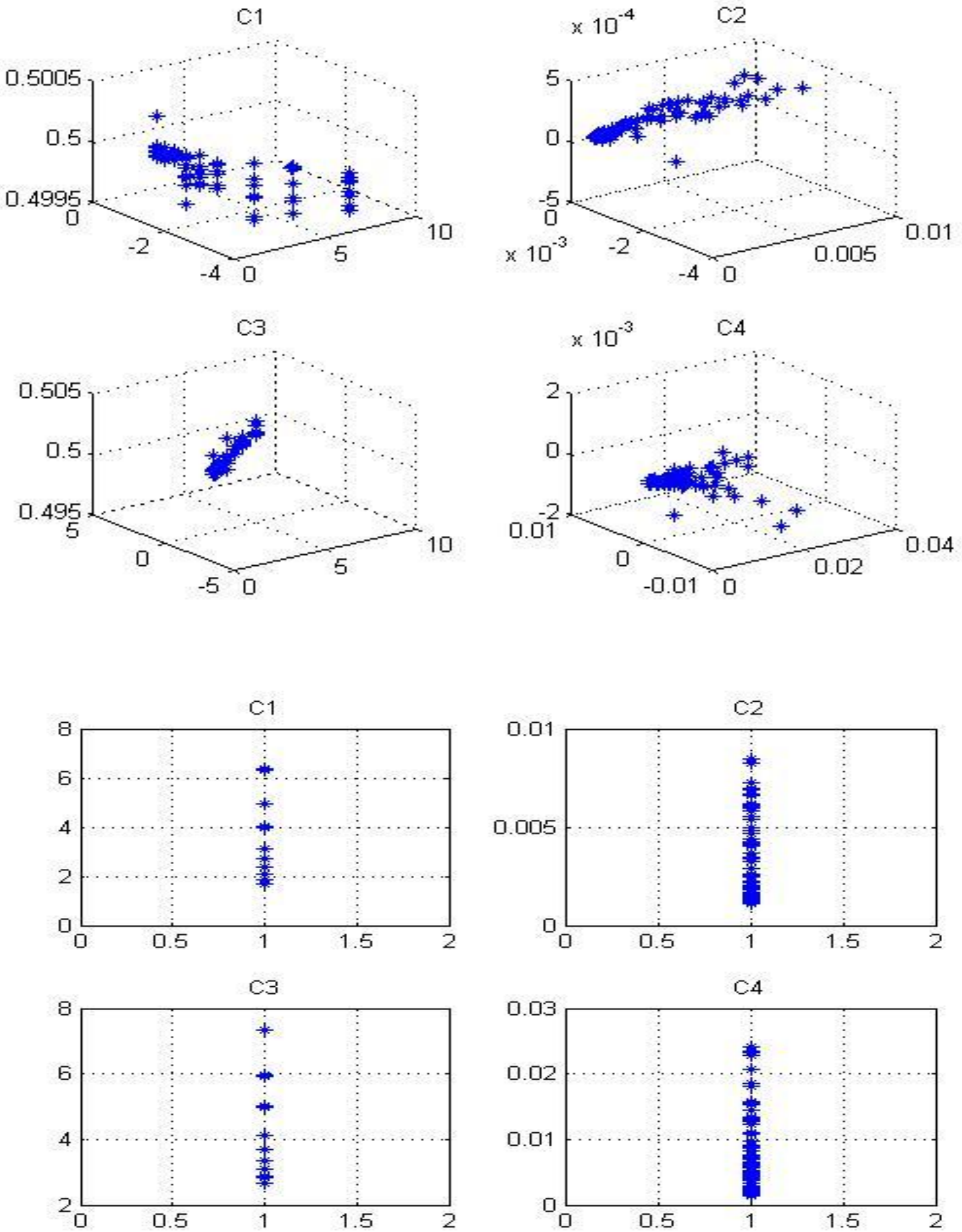

Figure (4.6) SDP – Uncertainty when (Δ1=0.79, Δ2=0.225, Δ3=0.363, Δ4=0.698)

We selected a random rate uncertainty values Δ for all the values that have been applied by the USDP and we see its acceptable at this time.

Chapter Five

Conclusion and Future Work

5.1 Conclusion

In this thesis, we proposed a new method to calculate the forces of dexterous hands for robotic systems. The proposed method depends on Linear programming and Semidefinite programming. In addition, we apply the uncertainty control system to measure the forces which make the system more robust.

Our proposed method, based on advance mathematic theorems such as Moore-Penrose pseudo-inverse, single value decomposition (SVD) ,and primary dual path algorithm.

Moore-Penrose pseudo-inverse has many advantages to compute a best fit solution to a system of linear equation that lacks a unique solution. In addition, finding the minimum norm solution to a system of linear equation with multiple solutions.

The applicability and effectiveness of the proposed method have been proven through solving many optimization examples and by comparing our results with other researchers were used different dextrose grasping methods as LP and SDP.

We apply the interval arithmetic – uncertainty LP and uncertainty SDP on the force matrices (C_1, C_2, C_3,C_4) and we get the upper bound and the lower bound of these forces which make the system more robust an stable and increasing the robustness will generally make the controller less aggressive.

5.2 Future Work

Searching in the field of dexterous hands for robotic systems is very rich and new. Moreover, the work in this thesis can be extended in many ways. Firstly, measuring the forces when the body is moving. Secondly, determining the forces of roughly objects, using another modern optimization techniques to solve the problems.

References

[1] M. Buss, H. Hashimoto, J. B. Moore, Dextrous Hand Grasping Force Optimization, *IEEE Transaction on Robotics Automation*. Vol. 12. NO 3. June 1996.

[2] S. K. Song, J. B. Park,Y. H. Choi, Stable Grasping Control Method of Dual-fingered Robot Hand for Force Angle Optimization and Position and Regulation, *American Control conference*, on O`Farrell Street, San Francisco, CA, USA, June 29 – July 01, 2011.

[3] Q. Zhan, R. Yang, Design and Optimization of a Robotic Finger, Beihang University, Beijing, China, 100191, *6th IEEE Conference on Industrial Electronics and Applications,* 2011.

[4] A. Sasaki, H. Hashimoto, S. Yokota, Y. Ohyama, C. Ishii, Force-distribution-based evaluation of product design suitable for dynamically dexterous human hand manipulation, Tokyo, Japan, 2011.

[5] H. Hashimoto, A. Sasaki, S. Yakota, Y. Ohyama, C. Ishii, Force distribution measurement to investigate dexterous manipulation of hand, Tokyo, Japan, *Proceeding of the IEEE International Conference on Mechatronics*, Istanbul, Turkey, April 13-15,2011.

[6] E. M. Al-Gallaf, A Learning Rule-Based Robotics Hand Optimal Force Closure, *Second International Conference on Computational Intelligence, Communication Systems and Networks*, University of Bahrain, Kingdom of Bahrain, 2010.

[7] T. Sugaiwa, G. Fujii, H. Iwata, S. Sugano, A Methodology for Setting Grasping Force for Picking up an Object with Unknown Weight, *Friction, and Stiffness, IEEE/RAS International Conference on Humanoid Robots Nashville*, TN, USA, December 6- 8, 2010.

[8] Y. Liu, H. Wang, B. Li, W. Zhou, An optimal method to determine the parameters of anthropomorphic finger based on four-link mechanism, *Proceedings of the 2nd Biennial IEEE/RAS-EMBS International Conference on Biomedical Robotics and Biomechatronics Scottsdale*, AZ, USA, October 19-22, 2008.

[9] A. Antoniou, W. S. Lu, Practical Optimization Algorithms and Engineering Applications, *Springer Science and Business Media LLC*, 2007.

[10] M. T. Hussein, Assessing 3-D Uncertain System Stability by Using MATLAB Convex Hull Functions, *(IJACSA) International Journal of Advanced Computer Science and Applications*, Electrical Engineering Department, Faculty of Engineering Islamic University of Gaza, Gaza Strip, vol. 2, no. 6, 2011.

[11] M. T. Hussein, An Efficient Computational Method For Bounds of Eigenvalues of Interval System Using A Convex Hull Algorithm, *The Arabian Journal for Science and Engineering*, vol. 35, no. 1B, pages 249-263, April 2010.

[12] M. T. Hussein, Lecture Notes for Control of Uncertain Systems Course Faculty of Engineering, Islamic University in Gaza, Jan- May 2010.

[13] A, Bicchi, Optimal control of robotic grasping, *in Proc. American Control Conference*, pp. 778-779, 1992.

[14] V. D. Nguyen. Constructing force-closure grasps, *in Proc. IEEE Inf. Conference Robotics and Automation*, pp. 1368-I373, 1986.

[15] V. D. Nguyen, The synthesis of stable grasps in the plane. *in Proc. IEEE Inf. Conf. Robotics and Automation, pp. 884-889*, 1986.

[16] V. D. Nguyen, Constructing stable grasps in 3d. *in Proc. IEEE Inf. Conf. Robotics and Automation*, pp. 234-239, 1987.

[17] D. J. Montana, The condition for contact grasp stability. *in Proc. IEEE Inf. Con& Robotics and Automation.* Sacramento, CA, pp. 412-417, 1991.

[18] C. H. Chen, K. W. Bosworth, M. P. Schoen, S. E. Bearden, D. S. Naidu, Fellow, A. Perez, A Study of Particle Swarm Optimization on Leukocyte Adhesion Molecules and Control Strategies for Smart Prosthetic Hand, *IEEE Swarm Intelligence Symposium St. Louis MO USA*, September 21-23, 2008.

[19] G. Chen, T. T. Pham, Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems, *CRC Press, Boca Raton London New York Washington*, Chapter one, 2001.

[20] Z. Xue, J. M. Zoellner, R. Dillmann, Automatic Optimal Grasp Planning Based On Found Contact Points, *Proceedings of the IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Xi'an, China, July 2 - 5, 2008.

[21] T. Watanabe, T. Yoshikawa, Fellow, Grasping Optimization Using a Required External Force Set, *IEEE Transaction on Automation Science and Engineering*, vol. 4, no. 1, 18 January 2007.

[22] L. Zollo, S. Roccella, E. Guglielmeli, M. C. Carrozza, P. Dario, Biomechatronic Design and Control of an Anthropomorphic Artificial Hand for Prosthetic and Robotic Applications, *IEEE/ASME Transactions on Mechatronics*, vol. 12, no. 4, August, 2007.

[23] K. Miyake, F. Nakamura, S. Komade, J. Hirai, Fast Decision of Contact Points of Multifingered Robot Hands using an Optimization Index of Fingertip Force, Mie University, *Proceeding of International Conference on Mechatronics*, Kumamoto Japan, 8-10 May 2007.

[24] Y. Zheng, W. H. Qian, A Fast Procedure for Optimizing Dynamic Force Distribution in Multifingered Grasping, *IEEE Transaction on Systems, Man, and Cybernetics-part B: Cybernetics*, vol. 36, no. 6, December 2006.

[25] Y. Xia, J. Wang, Lo-M. Fok, Grasping-Force Optimization for Multifingered Robotic Hands Using a Recurrent Neural Network, *IEEE Transactions on robotics and Automations*, vol. 20, no. 3, June 2004.

[26] C. S. Tzafestas, Whole-hand kinesthetic feedback and haptic perception in dextrous virtual manipulation, *IEEE Transaction on Systems, Man, and Cybernetics-part A: Systems and Humans*, vol. 33, no. 1, January 2003.

[27] H. Yoshida, K. Inoue, T. Arai, Y. Mae, Mobile Manipulation of Humanoid Robots Optimal Posture for Generating Large Force Based on Statics, *Proceedings of the IEEE International Conference on Robotics & Automation Washington*, DC, May 2002.

[28] T. Schlegl, M. Buss, T. Omata, G. Schmidt, Fast Dextrous Regrasping with Optimal Contact Forces and Contact Sensor-Based Impedance Control, *Proceedings of the IEEE International Conference on Robotics & Automation*, Seoul, Korea · May 21-26, 2001.

[29] C. Borst, M. Fischer, G. Hirzinger, A Fast and Robust Grasp Planner for Arbitrary 3D Objects, *Proceedings of the IEEE International Conference on Robotics & Automation*, Detroit, Michigan May 1999.

[30] J. Saut, C. Remond, V. Perdereau, M. Drouin, Online Computation of Grasping Force in Multi-fingered Hands, 1999.

[31] G. Guo, W. A. Gruver, Optimal Distribution of Fingertip Force Functions in Dynamic Grasping, 1999.

[32] F. T. Cheng, D. E. Orin, Optimal force distribution in multiple-chain robotic systems, *IEEE Transaction on Systems, Man, and Cybernetics*, vol. 21, no. 1, January / February 1991.

[33] C. S. Hwang, K. Sasaki, Control program of two fingered Dexterous Manipulation with Primitive motions, *Proceeding of the 2003 IEEE\RSJ, Conference on Intelligent Robots and System*, Las Vegas, Nevada, October 2003.

[34] H. Dobashi, A. Noda, Y. Yokokohji, H. Nagano, T. Nagatani, H. Okuda, Derivation of Optimal Robust Grasping Strategy under Initial Object Pose Errors, *The IEEE\RSJ International Conference on Intelligent Robots and System*, Taipei, Taiwan, October 18-22, 2010.

[35] B. Berret, S. Ivaldi, F. Nori, G. Sandini, Stochastic optimal control with variable impedance manipulators in presence of uncertainties and delayed feedback, *IEEE\RSJ International Conference on Intelligent Robots and System*, San Francisco, CA, USA, September 25-30, 2011.

[36] A. Ravindran, K. M. Ragsdell, G. V. Reklaitis, *Engineering Optimization Methods and Applications*, second edition, 2006.

[37] Z. Xue, M. Schmidt, J. M. Zoellner, R. Dillmann, Internal Force Computation Of Grasped Object Using Joint Torques, *SICE Annual Conference* ,The University Electro-Communications, Japan, , August 20-22, 2008.

[38] D. Zwillinger, S. G. Krantz, K. H. Rosen, CRC standard Mathematical Tables and formulae, *A CRC Press Company*, *Boca Raton London New York Washington*, 31st edition 2003.

Appendix A : Primary Dual Path Algorithm

Primal-dual path-following algorithm for SDP problems :

Step 1:

Input **A**i for $1 \leq i \leq p$, **b** $\in R^p$, **C** $\in R^{n\times n}$, and a strictly feasible set $\{\mathbf{X}_p, \mathbf{y}_0, \mathbf{S}_0\}$ that satisfies Eqs. $(A.1)$ and $(A.2)$ with $\mathbf{X}_0 \geq \mathbf{0}$ and $\mathbf{S}_0 \geq \mathbf{0}$.

$$subject\ to: \quad A_i.X = b_i \qquad (A.1)$$

For i = 1,2,3,..........P

$$subject\ to: \quad \sum y_i A_i + S = C \qquad (A.2)$$

Choose a scalar σ in the range $0 \leq \sigma < 1$.

Set k = 0 and initialize the tolerance ε for the duality gap δ_k.

Step 2:

Compute

$$\sigma_k = \frac{X_k S_k}{n} \qquad (A.3)$$

Step 3:

If $\delta_k \leq \varepsilon$, output solution $\{X_k, y_k, S_k\}$ and stop; otherwise, set

$$\tau_k = \sigma \frac{X_k S_k}{n} \qquad (A.4)$$

and continue with Step 4.

Step 4 :

Solve Eq. $(A.2)$ using Eqs. $(A.5)$–$(A.6)$ where $X = X_k$, $y = y_k$, $S = S_k$, and $\tau = \tau_k$.

Convert the solution $\{\Delta x, \Delta y, \Delta s\}$ into $\{\Delta X, \Delta y, \Delta S\}$ with $\Delta X = \text{mat}(\Delta x)$ and $\Delta S = \text{mat}(\Delta s)$.

Step 5 :

Choose a parameter γ in the range $0 < \gamma < 1$ and determine parameters α and β as

$$\alpha = \min(1, \gamma\hat{\alpha}) \qquad (A.5a)$$

$$\beta = \min\left(1, \gamma\hat{\beta}\right) \qquad (A.5b)$$

Where

$$\hat{\alpha} = \max_{X_k + \bar{\alpha}\Delta X \geq 0} (\bar{\alpha}) \qquad and \qquad \hat{\beta} = \max_{S_k + \bar{\beta}\Delta S \geq 0} (\bar{\alpha})$$

Step 6 :

Set

$$X_{k+1} = X_k + \alpha \Delta X \qquad (A.6a)$$

$$y_{k+1} = y_k + \beta \Delta y \qquad (A.6b)$$

$$S_{k+1} = S_k + \beta \Delta S \qquad (A.6c)$$

Set k = k + 1 and repeat from Step

From: **Wu-Sheng Lu** (wslu@ece.uvic.ca)

Sent: Thu 5/31/12 5:32 AM

To: raid alhabibi (raidalhabibi@hotmail.com)

Cc: Andreas Antoniou (aantoniou@shaw.ca)

Doctors E-mail Messages

Dear Raid,

Dr. Antoniou and I are glad that you found Practical Optimization useful in your research, thank you for your kind words .
We included the examples concerning optimal force distribution of multi-finger robotic manipulators in Chapter 16 of the text to indicate the potential use of contemporary optimization techniques in robotics. However, over a long time period in the past we have not been active in researching this field. It is therefore our advice that you should consult with IEEE Robotics and Automation community through its publications and conferences for current research topics and main-stream activities in the field.

With our best wishes,

Wu-Sheng Lu

On 5/13/2012 12:55 PM, raid alhabibi wrote:

Dear Professor . **Andreas Antoniou**

Dear Professor . **Wu-Sheng-Lu**

letter of thanks and gratitude

I would like to express my sincere thanks and gratitude for your massive efforts in supporting and helping us, I really appreciate your achievement in the electrical ,optimization ,control engineering.

your consultation helps us in implementing various projects in LP &SDP Optimal Force Distribution in Multi-finger Dexterous Hands for Robotic Systems and promote our capacity to develop the electricity sector .

Dr. Mohammed Hussein my professorial and supervisors at the masters and vice dean of Graduate studies – in IUG University he directed me to check the application for uncertainty control system in LP &SDP Optimal Force Distribution in Multi-finger Dexterous Hands for Robotic Systems after check the stability of the system and I would like to know what do you think about this field and what is your opinion about this project in thesis.

Again, thanks you so much for your help, I greatly appreciate the assistance you have provided us.

With best Regards,,,

Eng. Raed Abdul Rhman Al-Habibi

Raed Abdul Rhman Al-Habibi
Electrical & Power Engineering

Deputy Director of Projects, Technical Administration.
Gaza Electricity Distribution Corporation (GEDCO), Gaza, Palestine.
Tel. +970-8-2820221 begin_of_the_skype_highlighting +970-8-2820221 end_of_the_skype_highlighting , Fax. 2888481 , Mobile: 00970-599-171667begin_of_the_skype_highlighting 00970-599-171667 end_of_the_skype_highlighting

email :

raidalhabibi@hotmail.com

raedalhabibi@yahoo.com

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Deputy Director of Projects, Technical Administration.
Gaza Electricity Distribution Corporation (GEDCO), Gaza, Palestine.
Tel. +970-8-2820221 begin_of_the_skype_highlighting +970-8-2820221 end_of_the_skype_highlighting , Fax. 2888481 , Mobile: 00970-599-171667begin_of_the_skype_highlighting 00970-599-171667 end_of_the_skype_highlighting

email :

raidalhabibi@hotmail.com

raedalhabibi@yahoo.com

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To: raid alhabibi (raidalhabibi@hotmail.com)

Cc: Andreas Antoniou (aantoniou@shaw.ca)

Raid,

Dr. A. Antoniou has forwarded your email to me to respond as the expert on that subject concerning Eq. (16.85) and Example 16.7.

The material presented in Chapter 16 was built on many Chapters prior to Ch. 16. Specifically, understanding Eq. (16.85) and the solution of Example 16.7 require solid knowledge of (i) Sec. 10.4.1.1 and Appendix A.9; (ii) Sections 14.2, 14.3 and 14.4; and (iii) Sec. 16.4.1.

Enclosed are the MATLAB code ex16_7r.m and data file data_ex16_7r.mat that can be used to solve the SDP problem encountered in Example 16.7. However, we have to tell him that the above code will not work unless he has installed a MATLAB toolbox named "ROBUST CONTROL" TOOLBOX. This toolbox is a commercial product from the same company that produces MATLAB.

Sincerely,

Wu-Sheng Lu

From: **raid alhabibi** (raidalhabibi@hotmail.com)
Sent: Mon 4/09/12 9:24 PM
To: wslu@ece.uvic; raid alhabibi (raidalhabibi@hotmail.com)

Dear : Professor . **Wu-Sheng-Lu**

No one can deny your scientific contribution in Optimization Algorithms and Engineering Applications and you effort constructive in scientific fields ,What I would like to refer about me , I am Raed al Habibi I am working in GEDCO. And I am a robotics designer in IUG "Islamic university in Gaza" and i love your book and i study and teach it in our area , I am working and studying now optimal force distribution problem by using SDP and stop on the equation (16.85) and I hope you can help me to find how I can solve this example (16.7) and how we can get Φ* with mathematics by using By applying Algorithm 14.1 Primal dual path-following algorithm for SDP problems To

From this equation :

$$\text{minimize } \hat{p} = \hat{\mathbf{w}}^T \mathbf{\Phi}$$

$$\text{subject to: } \mathbf{P}(\mathbf{V}_\eta \Phi + \mathbf{c}0) \geq \mathbf{0}$$

how can we can solve matrix: with mathematic equation : "i hope you can send to me the full example solve about this example 16.7 from practical optimization algorithm & Enginnering Application Book page 568"

$$\mathbf{\Phi}^*$$

Best wishes

your student : Raid al Habibi

===

Raed Abdul Rhman Al-Habibi
Electrical & Power Engineering

Deputy Director of Projects, Technical Administration.
Gaza Electricity Distribution Corporation (GEDCO), Gaza, Palestine.
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how can we can solve matrix: with mathematic equation : "i hope you can send to me thefull solving about this example 16.7"

$$\mathbf{\Phi}^* =$$

Best wishes

your student : Raid al Habibi

==

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Electrical & Power Engineering

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Gaza Electricity Distribution Corporation (GEDCO), Gaza, Palestine.
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